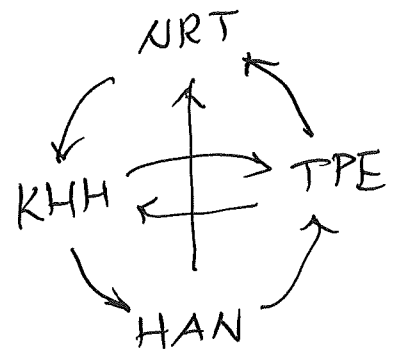


Graph theory : basics.

一組機票亂了, 排得回來嗎? (找一下起點 & 終點)

KHH \rightarrow TPE
TPE \rightarrow KHH
TPE \rightarrow NRT
KHH \rightarrow HAN

HAN \rightarrow NRT
NRT \rightarrow KHH
HAN \rightarrow TPE

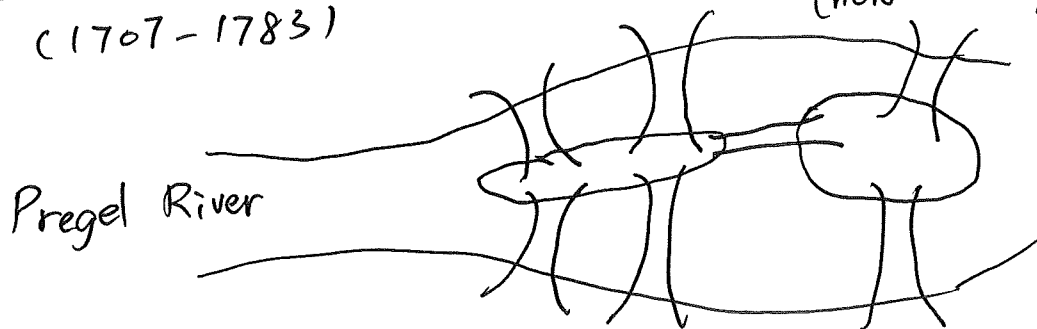


\Rightarrow 起: HAN, 終: NRT

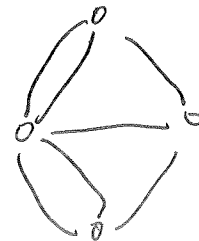
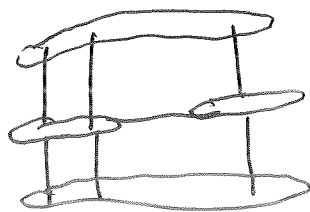
七橋問題: 能否每座橋各走一次, 回到原點?

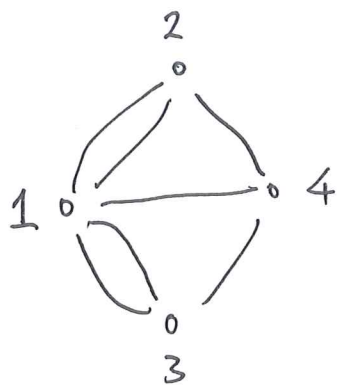
Leonhard Euler
(1707-1783)

Königsberg, Prussia
(now Kaliningrad, Russia)



一筆劃問題:





G
graph

vertices $V(G) = \{1, 2, 3, 4\}$.
(or nodes)

edges $E(G) = \{12, 12, 13, 13, 14, 24, 34, 34\}$.
點
邊

- two vertices i, j are adjacent if $ij \in E(G)$.
- the neighbors of vertex i is

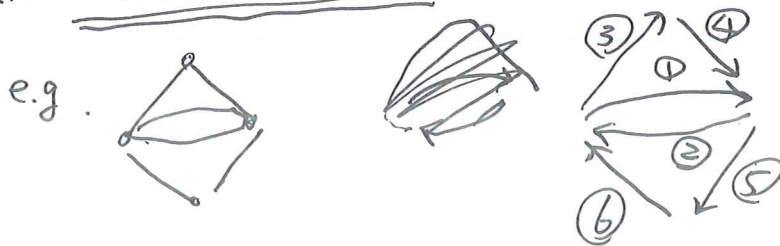
$$N(i) = \{j : ij \in E(G)\}.$$

e.g. $N(2) = \{1, 4\}$.

- the degree of vertex i is
 $\deg(i) = \text{number of edges on } i$.

e.g. $\deg(1) = 5$.

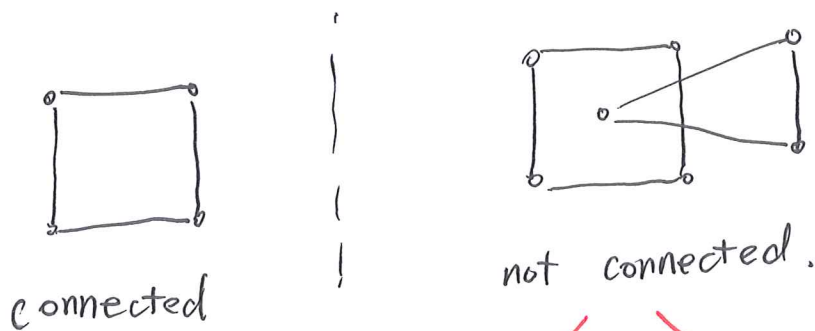
- an Eulerian circuit is a circuit using all edges.



Thm (Euler 1736)

A graph G has an Eulerian circuit
if and only if $\begin{cases} G \text{ is connected and} \\ \text{every vertex has even degree} \end{cases}$.

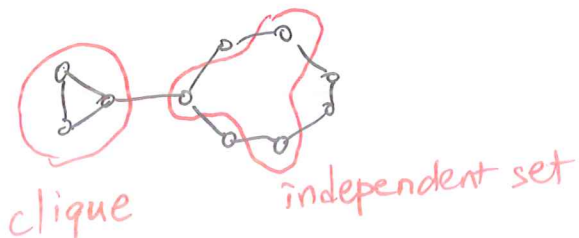
- A graph is connected if for any two vertices i, j , one may "walk" from i to j .



two connected components

Q: How to test if a graph is connected?

- A clique is a set of vertices such that any two vertices are adjacent.
- A independent set is a set of vertices such that any two vertices are not adjacent.



- A bipartite graph is a graph such that $V(G)$ has a partition (X, Y) and X, Y are independent sets.

