國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

MATH107A / GEAI1216A

數學導論（一） Introduction to Mathematics I

第一次期中考 October 17, 2025 **Midterm 1**

姓名 Name : ____________________

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **5 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. [1pt] State the definition of $\lim_{x \to c} f(x) = L$.

2. [1pt] State the negation of the definition of $\lim_{x \to c} f(x) = L$.

3. [1pt] Give a positive example and a negative example for the definition of $\lim_{x \to c} f(x) = L$. (You don't need to prove it.)

4. [1pt] State the definition of a linear combination.

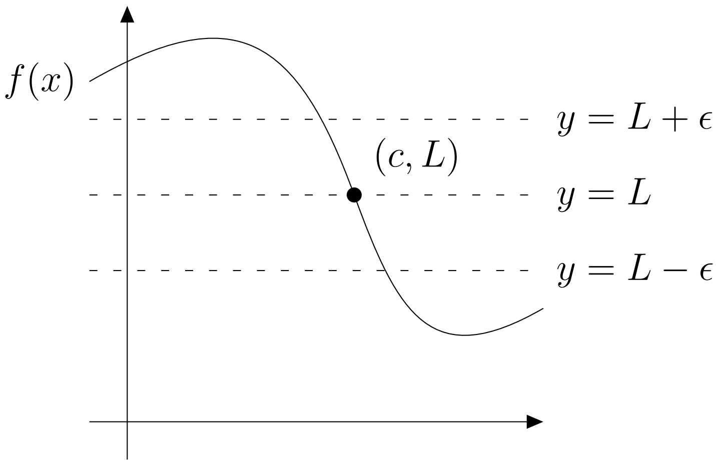
5. [1pt] Give a positive example and a negative example for the definition of a linear combination. (You don't need to prove it.)

 Every sentence should be complete.

6. [3pt] Prove that $\lim_{x\to 2} 3x^2 + 5 = 17$ using the ϵ-δ definition.

7. [2pt] Disprove that $\lim_{x\to 2} 3x^2 + 5 = 10$ using the ϵ-δ definition.

8. [5pt] Consider a function $f(x)$, a point (c, L), and $\epsilon > 0$ as shown in the picture.



Draw on the picture to demonstrate that you can find some $\delta > 0$ such that

$$0 < |x - c| < \delta \text{ implies } |f(x) - L| < \epsilon. \quad (1)$$

Then **explain carefully by words** why your idea is correct.

9. Let $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where

$$\begin{aligned} \mathbf{u}_1 &= (1, -2, 1, 0, 0), \\ \mathbf{u}_2 &= (0, 1, -2, 1, 0), \\ \mathbf{u}_3 &= (0, 0, 1, -2, 1). \end{aligned}$$

(a) [2pt] Prove that $\mathbf{p}_1 = (3, -2, 0, -6, 5)$ is a linear combination of S.

(b) [3pt] Prove that $\mathbf{p}_2 = (1, 2, 3, 4, 5)$ is not a linear combination of S.

10. [extra 2pt] Show that every point in $\mathbb{R}^3$ can be written as a linear combination of $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where

$$\mathbf{u}_1 = (1, 1, 1),$$
$$\mathbf{u}_2 = (1, 2, 4),$$
$$\mathbf{u}_3 = (1, 3, 9).$$

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	