國 立 陽 明 交 通 大 學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

微積分甲（一） **GECS10001**
Calculus A (I)

第一次期中考 October 16, 2026 **Midterm 1**

姓名 Name : solution

學號 Student ID # : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **4 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points**

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in **English**.

1. [1pt] Find $\lim_{x\to 3} \dfrac{x^2-5x+6}{x^2-2x-3}$.

$$= \lim_{x\to 3} \frac{(x-2)(x-3)}{(x-3)(x+1)} = \lim_{x\to 3} \frac{x-2}{x+1} = \frac{1}{4}$$
$$(x\neq 3)$$

2. [1pt] Find $\lim_{x\to\infty} \dfrac{2x^3}{3(x-1)^3}$.

$$= \lim_{x\to\infty} \frac{2}{3(1-\frac{1}{x})^3} = \frac{2}{3}$$

3. [1pt] Find $\lim_{\theta\to 0} \dfrac{\cos\theta - 1}{5\theta^2}$. [Hint: Multiply the function by $\frac{\cos\theta+1}{\cos\theta+1}$.]

$$= \lim_{\theta\to 0} \frac{\cos^2\theta - 1}{5\theta^2(\cos\theta+1)} = \lim_{\theta\to 0} \frac{-1}{5}\cdot\frac{\sin^2\theta}{\theta^2}\cdot\frac{1}{\cos\theta+1} = -\frac{1}{10}$$
$$\frac{\sin^2\theta}{\theta^2} \to 1$$

4. [1pt] Find $\lim_{x\to 0} \dfrac{|x|^3}{x}$.

$$\lim_{x\to 0^+} \frac{|x|^3}{x} = (x>0) \lim_{x\to 0^+} \frac{x^3}{x} = 0$$

$$\lim_{x\to 0^-} \frac{|x|^3}{x} = (x<0) \lim_{x\to 0^-} \frac{-x^3}{x} = 0$$

So the limit is 0.

5. [1pt] Find $\lim_{x\to 1} \dfrac{x^{5.5}-1}{x-1}$. [Note: You are NOT allowed to use L'Hôpital's rule if you have learned it. However, you may write the limit as the derivative of some function and use the power rule.]

By definition, $\lim_{x\to 1} \frac{x^{5.5}-1}{x-1} = f'(1)$ with $f(x) = x^{5.5}$

$$= 5.5\cdot x^{4.5}\Big|_{x=1}$$

$$= 5.5$$

6. [1pt] Let $f(x) = e^x \sin x$. Find $f'(x)$.

$f'(x) = (e^x)' \sin x + e^x (\sin x)'$

$= e^x \sin x + e^x \cos x$.

7. [1pt] Let $f(x) = \tan(x^2)$. Find $f'(x)$.

$f'(x) = \sec^2(x^2) \cdot 2x$

8. [2pt] Let $f(x) = \frac{1}{\sqrt{x}}$. Find the tangent line of $f(x)$ at $x = 1$.

Find slope :

$f'(1) = \left(x^{-\frac{1}{2}}\right)'\Big|_{x=1} = -\frac{1}{2} \cdot x^{-\frac{3}{2}}\Big|_{x=1} = -\frac{1}{2}$.

Suppose the line is $-\frac{1}{2}x + b$.

It passes through $(1, f(1)) = (1, 1)$, so $1 = -\frac{1}{2} + b$

$\Rightarrow b = \frac{3}{2}$

$\Rightarrow$ tangent line : $-\frac{1}{2}x + \frac{3}{2}$

9. [1pt] Fill the blanks for the definition of $\lim_{x \to a} f(x) = L$. (Each blank might contain more than one words.)

For every number $\epsilon > 0$, there is a number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

10. [5pt] Let

$$f(x) = -2x^3 + 21x^2 - 60x.$$

Sketch $f(x)$ and $f'(x)$. You need to include at least the following information:

- $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
- Mark at least two points of $f(x)$.
- Mark all points where $f'(x) = 0$.

x	-1	0	1
$f(x)$	83	0	-41

$f(1) = -2+21-60 = -41$

$f(-1) = 2+21+60 = 83$

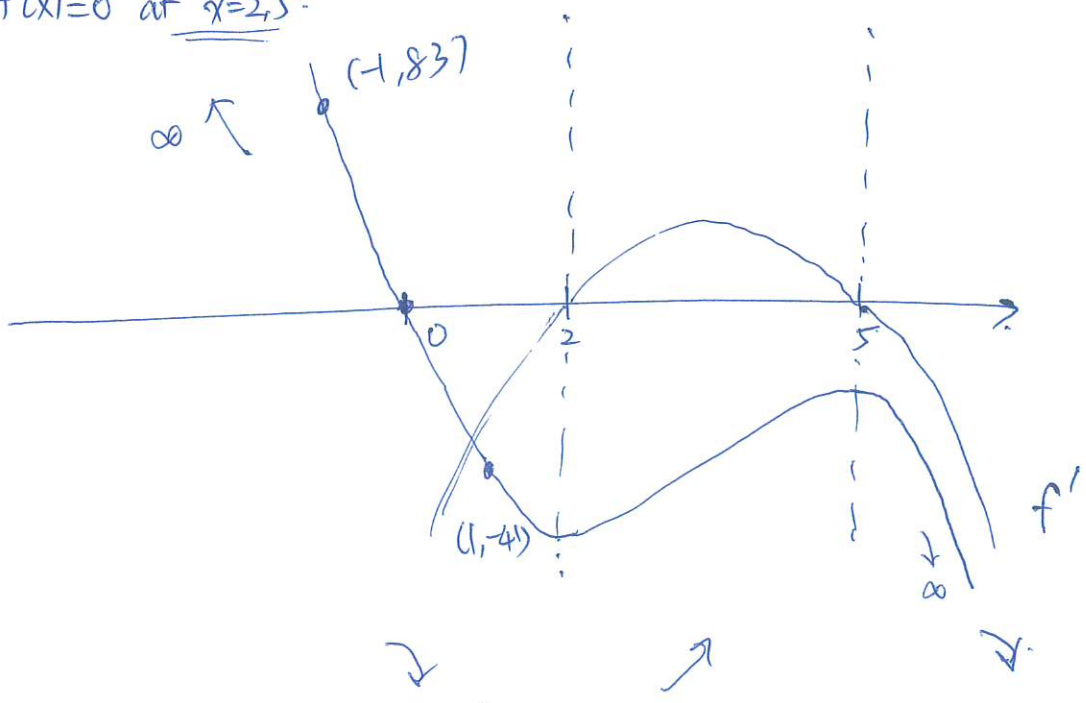
$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^3\left(-2+21\frac{1}{x}-60\frac{1}{x^2}\right) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

$f'(x) = -6x^2+42x-60 = -6(x^2-7x+10)$

$= -6(x-2)(x-5)$

$f'(x)=0$ at $x=2,5$.



11. [5pt] Let

$$f(x)=\begin{cases}\arctan(x+c)+1 & \text{if } x<-1,\\ 2 & \text{if } x=-1,\\ x^2 & \text{if } -1<x<1,\\ ax^2+bx-2 & \text{if } 1\le x.\end{cases}$$

(a) [1pt] Find c such that $\lim_{x\to -1} f(x)$ exists.

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} \arctan(x+c)+1 = \arctan(-1+c)+1$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} x^2 = 1$

limit exists $\Rightarrow \arctan(-1+c)+1 = 1 \Rightarrow \arctan(-1+c)=0 \Rightarrow c=1$

(b) [2pt] Find a, b such that $f'(1)$ exists.

$\lim_{h\to 0^-} \frac{f(x+h)-f(x)}{h} = (x^2)'\Big|_{x=1} = 2$

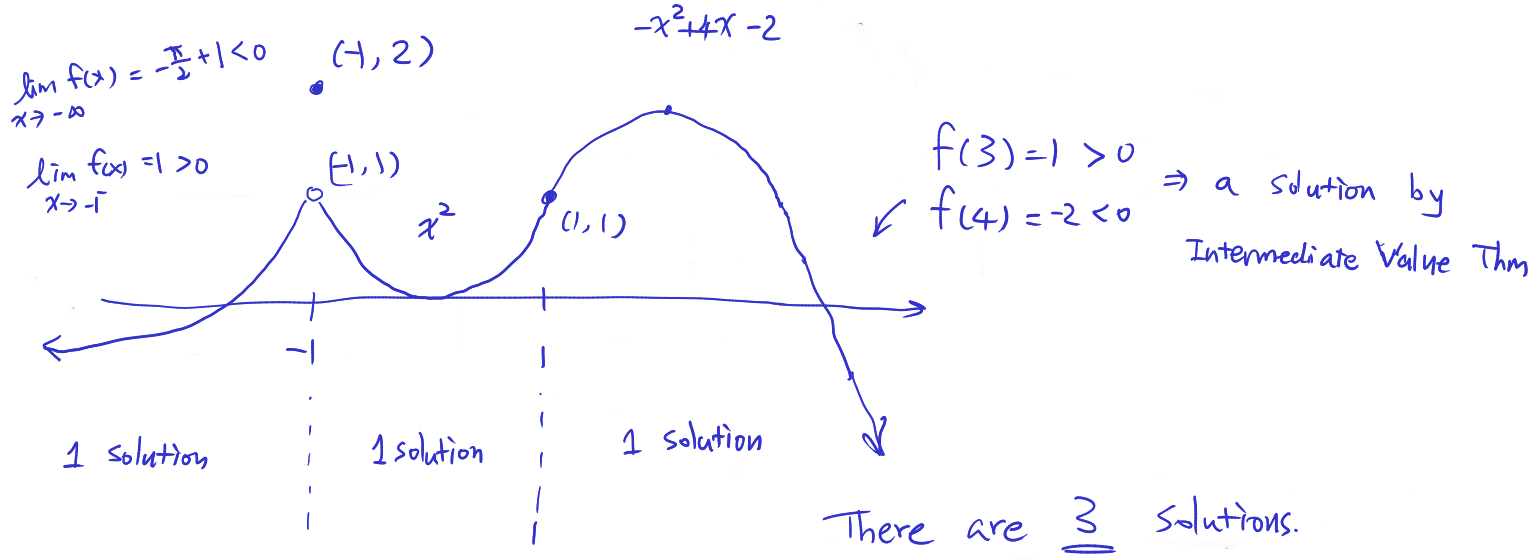
$\Rightarrow 2a+b=2$

$\lim_{h\to 0^+} \frac{f(x+h)-f(x)}{h} = (ax^2+bx-2)'\Big|_{x=1} = (2ax+b)\Big|_{x=1} = 2a+b$

Also, $f'(1)$ exists means $\lim_{x\to 1} f(x) = f(1)$, so $1 = a+b-2$

$\begin{cases} 2a+b=2 \\ a+b=3 \end{cases} \Rightarrow \begin{cases} a=-1 \\ b=4 \end{cases}$

(c) [2pt] How many solutions does the equation $f(x)=0$ have? Explain your reasons.



[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20	