國 立 陽 明 交 通 大 學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

微積分甲（一） **GECS10001**
Calculus A (I)

第一次期中考 October 16, 2026 **Midterm 1**

姓名 Name : Solution

學號 Student ID # : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **4 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points**

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in **English**.

1. [1pt] Find $\lim_{x\to 3} \dfrac{x^2-x-6}{x^2-2x-3}$.

$= \lim_{x\to 3} \dfrac{(x-3)(x+2)}{(x-3)(x+1)} \overset{(x\neq 3)}{=} \lim_{x\to 3} \dfrac{x+2}{x+1} = \dfrac{5}{4}$

2. [1pt] Find $\lim_{x\to\infty} \dfrac{3x^3}{4(x-1)^3}$.

$= \lim_{x\to\infty} \dfrac{3}{4(1-\frac{1}{x})^3} = \dfrac{3}{4}$

3. [1pt] Find $\lim_{\theta\to 0} \dfrac{\cos\theta - 1}{3\theta^2}$. [Hint: Multiply the function by $\frac{\cos\theta+1}{\cos\theta+1}$.]

$= \lim_{\theta\to 0} \dfrac{\cos^2\theta - 1}{3\theta^2\cdot(\cos\theta+1)} = \lim_{\theta\to 0} \dfrac{1}{3}\cdot\dfrac{\sin^2\theta}{\theta^2}\cdot\dfrac{1}{\cos\theta+1} = -\dfrac{1}{6}$

4. [1pt] Find $\lim_{x\to 0} \dfrac{x^3}{|x|}$.

$\lim_{x\to 0^+} \dfrac{x^3}{|x|} = \lim_{x\to 0^+} \dfrac{x^3}{x} = 0$

$\lim_{x\to 0^-} \dfrac{x^3}{|x|} = \lim_{x\to 0^-} \dfrac{x^3}{-x} = 0$

So the limit is 0.

5. [1pt] Find $\lim_{x\to 1} \dfrac{x^{7.5}-1}{x-1}$. [Note: You are NOT allowed to use L'Hôpital's rule if you have learned it. However, you may write the limit as the derivative of some function and use the power rule.]

By definition, $\lim_{x\to 1} \dfrac{x^{7.5}-1}{x-1} = f'(1)$ with $f(x) = x^{7.5}$

$= 7.5\cdot x^{6.5}\Big|_{x=1}$

$= 7.5$

6. [1pt] Let $f(x) = e^x \cos x$. Find $f'(x)$.

$f'(x) = (e^x)' \cos x + e^x (\cos x)'$

$= e^x \cos x - e^x \sin x$.

7. [1pt] Let $f(x) = \sec(x^2)$. Find $f'(x)$.

$f'(x) = \tan(x^2) \sec(x^2) \cdot 2x$

8. [2pt] Let $f(x) = \sqrt{x}$. Find the tangent line of $f(x)$ at $x = 1$.

$f(x) = x^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$.

Find slope $f'(1) = \frac{1}{2}$.

Find tangent line $\frac{1}{2}x + b$ passing through $(1, f(1)) = (1, 1)$

$\frac{1}{2} + b = 1 \Rightarrow b = \frac{1}{2}$.

So tangent line is $\frac{1}{2}x + \frac{1}{2}$.

9. [1pt] Fill the blanks for the definition of $\lim_{x \to a} f(x) = L$. (Each blank might contain more than one words.)

For every number $\epsilon > 0$, there is a number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

10. [5pt] Let

$$f(x) = -2x^3 + 21x^2 - 60x.$$

Sketch $f(x)$ and $f'(x)$. You need to include at least the following information:

- $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
- Mark at least two points of $f(x)$.
- Mark all points where $f'(x) = 0$.

See ver. A.

11. [5pt] Let

$$f(x) = \begin{cases} \arctan(x+c)+1 & \text{if } x < -1, \\ 2 & \text{if } x = -1, \\ x^2 & \text{if } -1 < x < 1, \\ ax^2 + bx - 2 & \text{if } 1 \leq x. \end{cases}$$

(a) [1pt] Find c such that $\lim_{x \to -1} f(x)$ exists.

See ver. A.

(b) [2pt] Find a, b such that $f'(1)$ exists.

(c) [2pt] How many solutions does the equation $f(x) = 0$ have? Explain your reasons.

[END]

Page	Points	Score
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3	5	
4	5	
5	2	
Total	20	