國 立 陽 明 交 通 大 學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

Python 與機器學習演算法 **SCMA30084**
Python and Machine Learning Algorithms

第一次段考 March 30, 2026 **Exam 1**

姓名 Name : ____________________

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**180 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

– Enter your **Name** and **Student ID #** before you start.

– If the answer is too long, round it to **two decimal places**. Numerical answers with a **tolerance ±0.05** are accepted.

– Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

– Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

– Please answer the problems in **English**.

1. Define the NumPy arrays as follows.

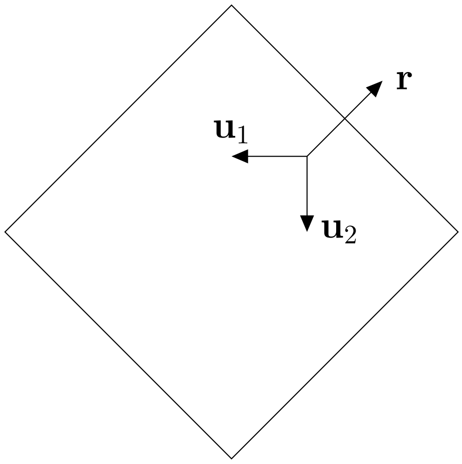
```
arr1 = np.array([1,2,3])
arr2 = np.array([[1,2,3]])
arr3 = np.array([[1],[2],[3]])
arr4 = np.array([[1,2],[2,3]])
```

   (a) [1pt] For each array above, write down its shape.

   (b) [2pt] What is the output of `(arr2 - 1) * arr3`? Describe the broadcasting rule.

   (c) [2pt] What is the output of `arr4 - arr4.mean(axis=0)`? Describe the broadcasting rule.

2. Consider the hyperplane with its normal vector $\mathbf{r}$ as shown below. Let $\mathbf{u}_1$ and $\mathbf{u}_2$ be vectors falling on the hyperplane.



$$\mathbf{r} = (-1, -1, 1, 1)$$
$$\mathbf{u}_1 = (1, 1, 1, 1)$$
$$\mathbf{u}_2 = (1, -1, -1, 1)$$

(a) [2pt] For each of the following points, **draw** it on the picture above to demonstrate its relative position with the hyperplane (on the hyperplane, on the same side as $\mathbf{r}$, or on the opposite side) and **find** its projection onto the the direction of $\mathbf{r}$.

$$\mathbf{p}_1 = (2, 2, 1, 1),$$
$$\mathbf{p}_2 = (2, 1, 2, 1),$$
$$\mathbf{p}_3 = (2, 1, 1, 2).$$

(b) [3pt] Let A be the matrix whose columns are $\mathbf{u}_1$ and $\mathbf{u}_2$ in order. Draw

$$A\begin{bmatrix}3\\1\end{bmatrix}, \quad A\begin{bmatrix}1\\3\end{bmatrix}, \text{ and } \mathbf{q} = (4, 0, 0, 4)$$

on the picture above.

3. Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}.$$

(a) [1pt] Find the eigenvalues of A and an orthonormal eigenbasis of it.

(b) [1pt] Let $R_A(\mathbf{x}) = \frac{\mathbf{x}^\top A \mathbf{x}}{\mathbf{x}^\top \mathbf{x}}$. Solve

$$\min_{\mathbf{x} \neq \mathbf{0}} R_A(\mathbf{x})$$

and find a vector $\mathbf{x}$ that achieves the minimum.

(c) [1pt] Find a vector $\mathbf{x}$ such that $R_A(\mathbf{x}) = 0$. Describe your reasoning.

(d) [2pt] Apply the Power Method to A using the initial vector $\mathbf{x}_0 = (1, -1, 0, 0)$. Which vector does $\mathbf{x}_k$ converge to? Explain why $\mathbf{x}_k$ does not converge to the eigenvector corresponding to the largest eigenvalue.

4. [5pt] Mathematical essay: Write a few paragraphs to introduce the *spectral decomposition*.

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

5. [extra 2pt] Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 2 \end{bmatrix}.$$

Find the vector $\mathbf{x}$ such that $\|A\mathbf{x} - \mathbf{b}\|^2$ is minimized.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	