國 立 陽 明 交 通 大 學
NATIONAL YANG MING CHIAO TUNG UNIVERSITY

Python 與機器學習演算法 **SCMA30084**
Python and Machine Learning Algorithms

第一次段考 March 30, 2026 **Exam 1**

姓名 Name : solution

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **5 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**180 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- If the answer is too long, round it to **two decimal places**. Numerical answers with a **tolerance** $\pm$**0.05** are accepted.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in **English**.

1. Define the NumPy arrays as follows.

```
arr1 = np.array([1,2,3])
arr2 = np.array([[1,2,3]])
arr3 = np.array([[1],[2],[3]])
arr4 = np.array([[1,2],[2,3]])
```

(a) [1pt] For each array above, write down its shape.

arr1 → ~~(3,1)~~ (3,)

arr2 → (1,3)

arr3 → (3,1)

arr4 → (2,2)

(b) [2pt] What is the output of `(arr2 - 1) * arr3`? Describe the broadcasting rule.

$$arr2 - 1 = [1\ 2\ 3] - 1 \overset{\text{broadcasting}}{=} \cancel{[0,\ 1,\ 2]}\ [1\ 2\ 3] - [1\ 1\ 1]$$

$$= [0\ 1\ 2]$$

$$(arr2 - 1) * arr3 = [0\ 1\ 2] * \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \overset{\text{broadcasting}}{=} \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 2 & 4 \\ 0 & 3 & 6 \end{bmatrix}$$

(c) [2pt] What is the output of `arr4 - arr4.mean(axis=1)`? Describe the broadcasting rule.

0

arr4.mean(axis=0) = average of each column of $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

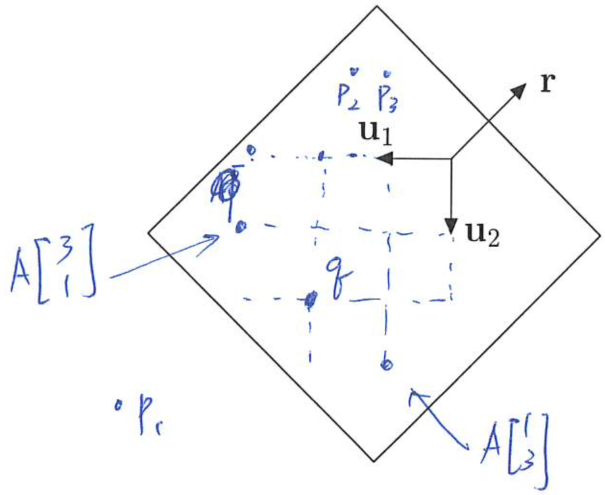
$$= \left[\frac{1+2}{2}\ \ \frac{2+3}{2}\right] = [1.5\ \ 2.5]$$

$$arr4 - arr4.mean(axis=0) = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} - [1.5\ \ 2.5]$$

$$\overset{\text{broadcasting}}{=} \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1.5 & 2.5 \\ 1.5 & 2.5 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

2. Consider the hyperplane with its normal vector $\mathbf{r}$ as shown below. Let $\mathbf{u}_1$ and $\mathbf{u}_2$ be vectors falling on the hyperplane.



$$\mathbf{r} = (-1,-1,1,1)$$
$$\mathbf{u}_1 = (1,1,1,1)$$
$$\mathbf{u}_2 = (1,-1,-1,1)$$

(a) [2pt] For each of the following points, **draw** it on the picture above to demonstrate its relative position with the hyperplane (on the hyperplane, on the same side as $\mathbf{r}$, or on the opposite side) and **find** its projection onto the the direction of $\mathbf{r}$.

$\langle P_1, r\rangle = -2 \rightarrow$ oppo side of $\vec{r}$ $\quad \mathbf{p}_1 = (2,2,1,1),$ $\quad$ projection formula

$\langle P_2, r\rangle = 0 \rightarrow$ on the hyperplane $\quad \mathbf{p}_2 = (2,1,2,1),$ $\quad \dfrac{\langle P, r\rangle}{\|r\|^2}\cdot r$

$\langle P_3, r\rangle = 0 \rightarrow$ $\quad \mathbf{p}_3 = (2,1,1,2).$

proj of $P_1 = \dfrac{-2}{4}\cdot\begin{bmatrix}-1\\-1\\1\\1\end{bmatrix} = -\dfrac{1}{2}\begin{bmatrix}-1\\-1\\1\\1\end{bmatrix}$

proj of P_2 = proj of $P_3 = \vec{0}$

(b) [3pt] Let A be the matrix whose columns are $\mathbf{u}_1$ and $\mathbf{u}_2$ in order. Draw

$$A\begin{bmatrix}3\\1\end{bmatrix}, \quad A\begin{bmatrix}1\\3\end{bmatrix}, \text{ and } \mathbf{q} = (4,0,0,4)$$

on the picture above.

$A = \begin{bmatrix} | & | \\ u_1 & u_2 \\ | & | \end{bmatrix}$ $\quad$ So $A\begin{bmatrix}3\\1\end{bmatrix} = 3u_1 + 1u_2$

$A\begin{bmatrix}1\\3\end{bmatrix} = 1u_1 + 3u_2$

Solve $q = 2u_1 + 2u_2$.

3. Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}.$$

(a) [1pt] Find the eigenvalues of A and an orthonormal eigenbasis of it.

(b) [1pt] Let $R_A(\mathbf{x}) = \frac{\mathbf{x}^\top A\mathbf{x}}{\mathbf{x}^\top \mathbf{x}}$. Solve

$$\min_{\mathbf{x}\neq\mathbf{0}} R_A(\mathbf{x})$$

and find a vector $\mathbf{x}$ that achieves the minimum.

(c) [~~2~~1pt] Find a vector $\mathbf{x}$ such that $R_A(\mathbf{x}) = 0$. Describe your reasoning.

(d) [2pt] Apply the Power Method to A using the initial vector $\mathbf{x}_0 = (1, -1, 0, 0)$. Which vector does $\mathbf{x}_k$ converge to? Explain why $\mathbf{x}_k$ does not converge to the eigenvector corresponding to the largest eigenvalue, and discuss how likely this situation is to occur.

See ver A.

4. [5pt] Mathematical essay: Write a few paragraphs to introduce the *spectral decomposition.*

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

5. [extra 2pt] Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 2 \end{bmatrix}.$$

Find the vector $\mathbf{x}$ such that $\|A\mathbf{x} - \mathbf{b}\|^2$ is minimized.

See ver A.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	