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Nonlinear Schrödinger Solitary Waves

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Introduction

We consider that nonlinear Schrödinger (NLS) equations with focusing power nonlinearities have solitary wave solutions [1, 3, 4].

$$i\partial_t\psi = -\Delta\psi - |\psi|^{p-1}\psi, \qquad (1)$$

where $\psi \doteq \psi(t,x)$ is a complex function, x is a n-dimensional real variable, and the nonlinearity power p, $1 < p < \infty$.

To study the spectra of the "linearized operators" which arise when the NLS (1) is linearized around the solitary waves.

Case 1: Non-trivial radial $\psi(t,x) = \phi(r)\, e^{it}$.

Case 2: Non-radial $\psi(t,x) = \phi(r)\, e^{im\theta}\, e^{it}$ (non-zero angular momenta sol.).

The spectra of the linearized operators around these solitary waves are intimately connected to stability properties of the solitary waves, and to the long-time dynamics of solutions of (NLS).

Conclusions

We observe a more detailed understanding of the spectrum of $\mathcal{L}$, using numerical techniques [2].

- Estimate the number and locations of the ew.s of the linearized operator $\mathcal{L}$.
- Bifurcations, as p varies, of pairs of purely imaginary ew.s into pairs of ew.s with non-zero real part (a stability/instability transition, see Fig. 1).

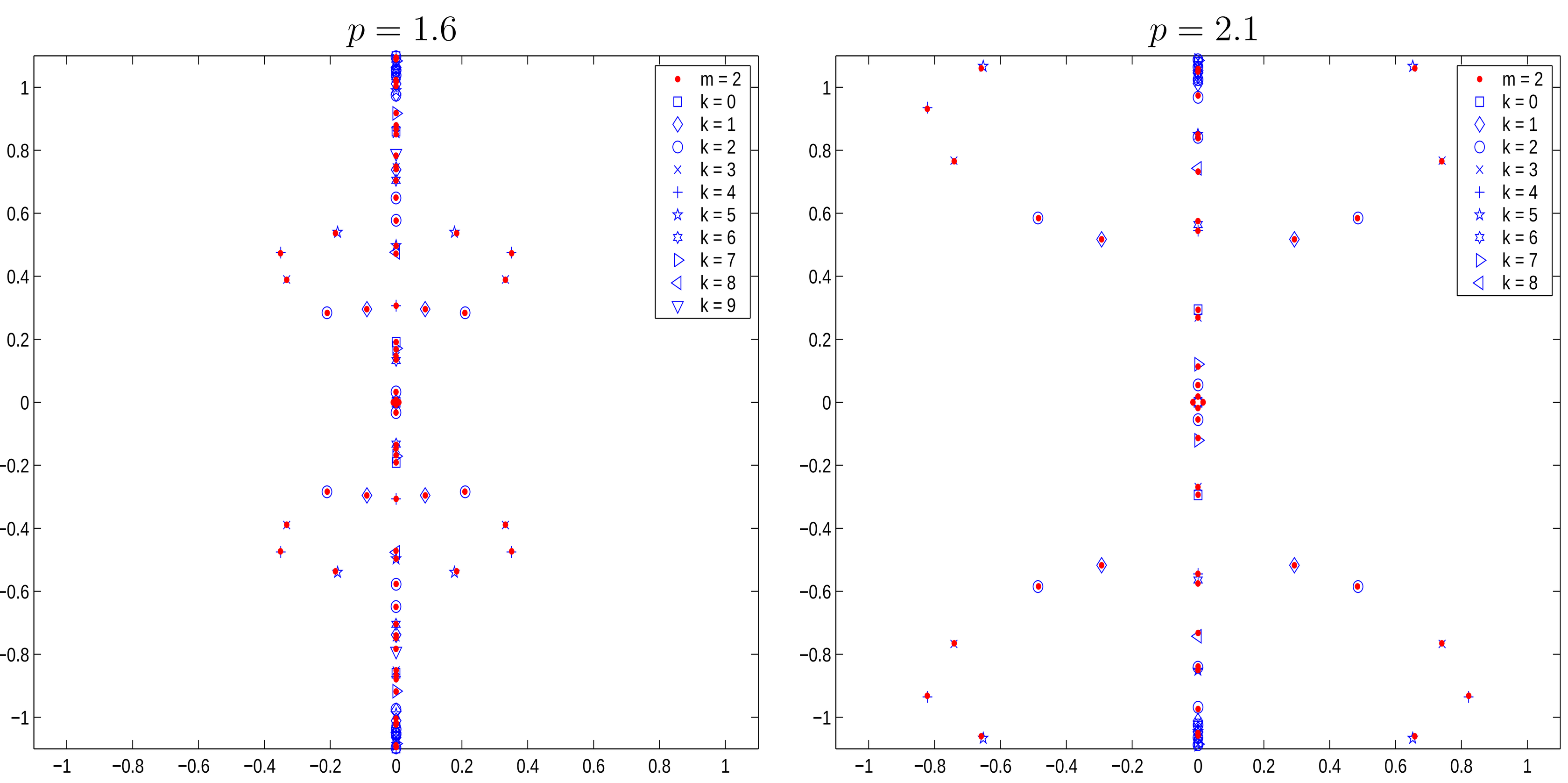

Fig. 1: Spectra of $\mathcal{L}$ for $n, m = 2$ and various $p = 1.6, 2.1$. "·" denotes the spectra computed by Alg. 1, and the other blue symbols denote the spectra computed by Alg. 2 and 3.

Mathematical Model

The linearized operators come from $\psi(t,x) = [Q(x) + h(t,x)]\, e^{it}$ and the NLS (1).

Case 1: Non-trivial radial $\mathcal{L}h = -i\left\{(-\Delta + 1 - Q^{p-1})h - \frac{p-1}{2} Q^{p-1}(h + \bar{h})\right\}$;

Case 2: Non-radial $\mathcal{L}h = i\left(\Delta h - h + \frac{p+1}{2}|Q|^{p-1}h + \frac{p-1}{2}|Q|^{p-3}Q^2\bar{h}\right)$.

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