

**A COMBINATORIAL PROOF OF THE CYCLIC SIEVING PHENOMENON  
FOR FACES OF COXETERHEDRA**

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For a Coxeter system  $(W, S)$ , the subgroups  $W_J$  generated by subsets  $J \subseteq S$  are called *parabolic subgroups* of  $W$ . The *Coxeterhedron*  $PW$  associated to  $(W, S)$  is the finite poset of all cosets  $\{wW_J\}_{w \in W, J \subseteq S}$  of all parabolic subgroups of  $W$ , ordered by inclusion. This poset can be realized by the face lattice of a simple polytope, constructed as the convex hull of the orbit of a generic point in  $\mathbb{R}^n$  under an action of the reflection group  $W$ . For the groups  $W = A_{n-1}, B_n$ , and  $D_n$  in a case-by-case manner, we present an elementary proof of the cyclic sieving phenomenon (CSP) for faces of various dimensions of  $PW$  under the action of a cyclic group generated by a Coxeter element. This result provides a geometric, enumerative and combinatorial approach to the classical type of a theorem in [Reiner-Stanton-White, The cyclic sieving phenomenon, J. Combinatorial Theory Ser. A 108 (2004) 17–50], which is proved by an algebraic method that involves representation theory and Springer’s theorem on regular elements. In this talk, we shall give a brief introduction on the notion CSP and present the combinatorial and algebraic aspects of the CSP for faces of Coxeterhedra. This talk is based on joint work with S.-P. Eu and Y.-J. Pan.