1. (a) From Table 1, $f(-15, 40) = -27$, which means that if the temperature is $-15°C$ and the wind speed is 40 km/h, then the air would feel equivalent to approximately $-27°C$ without wind.

(b) The question is asking: when the temperature is $-20°C$, what wind speed gives a wind-chill index of $-30°C$? From Table 1, the speed is 20 km/h.

(c) The question is asking: when the wind speed is 20 km/h, what temperature gives a wind-chill index of $-49°C$? From Table 1, the temperature is $-35°C$.

(d) The function $W = f(-5, v)$ means that we fix T at -5 and allow v to vary, resulting in a function of one variable. In other words, the function gives wind-chill index values for different wind speeds when the temperature is $-5°C$. From Table 1 (look at the row corresponding to $T = -5$), the function decreases and appears to approach a constant value as v increases.

(e) The function $W = f(T, 50)$ means that we fix v at 50 and allow T to vary, again giving a function of one variable. In other words, the function gives wind-chill index values for different temperatures when the wind speed is 50 km/h . From Table 1 (look at the column corresponding to $v = 50$), the function increases almost linearly as T increases.

2. (a) From Table 3, $f(95, 70) = 124$, which means that when the actual temperature is $95°F$ and the relative humidity is 70%, the perceived air temperature is approximately $124°F$.

(b) Looking at the row corresponding to $T = 90$, we see that $f(90, h) = 100$ when $h = 60$.

(c) Looking at the column corresponding to $h = 50$, we see that $f(T, 50) = 88$ when $T = 85$.

(d) $I = f(80, h)$ means that T is fixed at 80 and h is allowed to vary, resulting in a function of h that gives the humidex values for different relative humidities when the actual temperature is $80°F$. Similarly, $I = f(100, h)$ is a function of one variable that gives the humidex values for different relative humidities when the actual temperature is $100°F$. Looking at the rows of the table corresponding to $T = 80$ and $T = 100$, we see that $f(80, h)$ increases at a relatively constant rate of approximately $1°F$ per 10% relative humidity, while $f(100, h)$ increases more quickly (at first with an average rate of change of $5°F$ per 10% relative humidity) and at an increasing rate (approximately $12°F$ per 10% relative humidity for larger values of h).

3. If the amounts of labor and capital are both doubled, we replace L, K in the function with $2L, 2K$, giving

$$P(2L, 2K) = 1.01(2L)^{0.75}(2K)^{0.25} = 1.01(2^{0.75})(2^{0.25})L^{0.75}K^{0.25} = (2^1)1.01L^{0.75}K^{0.25} = 2P(L, K)$$

Thus, the production is doubled. It is also true for the general case $P(L, K) = bL^{\alpha}K^{1-\alpha}$:

$P(2L, 2K) = b(2L)^{\alpha}(2K)^{1-\alpha} = b(2^{\alpha})(2^{1-\alpha})L^{\alpha}K^{1-\alpha} = (2^{\alpha+1-\alpha})bL^{\alpha}K^{1-\alpha} = 2P(L, K)$.

4. We compare the values for the wind-chill index given by Table 1 with those given by the model function:

Modeled Wind-Chill Index Values $W(T, v)$

Wind Speed (km/h)

Actual temperature (°C)

T \ V	5	10	15	20	25	30	40	50	60	70	80
5	4.08	2.66	1.74	1.07	0.52	0.05	−0.71	−1.33	−1.85	−2.30	−2.70
0	−1.59	−3.31	−4.42	−5.24	−5.91	−6.47	−7.40	−8.14	−8.77	−9.32	−9.80
−5	−7.26	−9.29	−10.58	−11.55	−12.34	−13.00	−14.08	−14.96	−15.70	−16.34	−16.91
−10	−12.93	−15.26	−16.75	−17.86	−18.76	−19.52	−20.77	−21.77	−22.62	−23.36	−24.01
−15	−18.61	−21.23	−22.91	−24.17	−25.19	−26.04	−27.45	−28.59	−29.54	−30.38	−31.11
−20	−24.28	−27.21	−29.08	−30.48	−31.61	−32.57	−34.13	−35.40	−36.47	−37.40	−38.22
−25	−29.95	−33.18	−35.24	−36.79	−38.04	−39.09	−40.82	−42.22	−43.39	−44.42	−45.32
−30	−35.62	−39.15	−41.41	−43.10	−44.46	−45.62	−47.50	−49.03	−50.32	−51.44	−52.43
−35	−41.30	−45.13	−47.57	−49.41	−50.89	−52.14	−54.19	−55.84	−57.24	−58.46	−59.53
−40	−46.97	−51.10	−53.74	−55.72	−57.31	−58.66	−60.87	−62.66	−64.17	−65.48	−66.64

The values given by the function appear to be fairly close (within 0.5) to the values in Table 1.

5. (a) According to Table 4, $f(40, 15) = 25$, which means that if a 40-knot wind has been blowing in the open sea for 15 hours, it will create waves with estimated heights of 25 feet.

(b) $h = f(30, t)$ means we fix v at 30 and allow t to vary, resulting in a function of one variable. Thus here, $h = f(30, t)$ gives the wave heights produced by 30-knot winds blowing for t hours. From the table (look at the row corresponding to $v = 30$), the function increases but at a declining rate as t increases. In fact, the function values appear to be approaching a limiting value of approximately 19, which suggests that 30-knot winds cannot produce waves higher than about 19 feet.

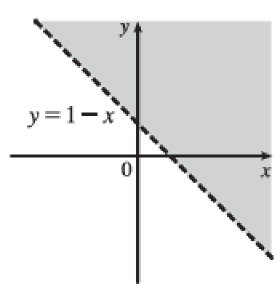
(c) $h = f(v, 30)$ means we fix t at 30, again giving a function of one variable. So, $h = f(v, 30)$ gives the wave heights produced by winds of speed v blowing for 30 hours. From the table (look at the column corresponding to $t = 30$), the function appears to increase at an increasing rate, with no apparent limiting value. This suggests that faster winds (lasting 30 hours) always create higher waves.

6. (a) $f(1, 1) = \ln(1 + 1 - 1) = \ln 1 = 0$

(b) $f(e, 1) = \ln(e + 1 - 1) = \ln e = 1$

(c) $\ln(x + y - 1)$ is defined only when $x + y - 1 > 0$, that is, $y > 1 - x$. So the domain of f is $\{(x, y) \mid y > 1 - x\}$.

(d) Since $\ln(x + y - 1)$ can be any real number, the range is $\mathbb{R}$.

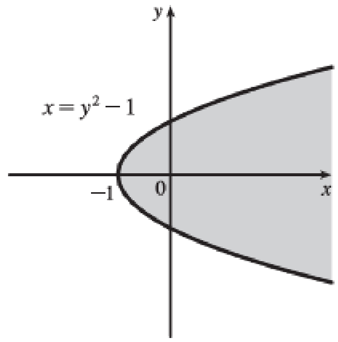


7. (a) $f(2,0)=2^2e^{3(2)(0)}=4(1)=4$

(b) Since both x^2 and the exponential function are defined everywhere, x^2e^{3xy} is defined for all choices of values for x and y. Thus the domain of f is $\mathbb{R}^2$.

(c) Because the range of $g(x,y)=3xy$ is $\mathbb{R}$, and the range of e^x is $(0,\infty)$, the range of $e^{g(x,y)}=e^{3xy}$ is $(0,\infty)$. The range of x^2 is $[0,\infty)$, so the range of the product x^2e^{3xy} is $[0,\infty)$.

8. $\sqrt{1+x-y^2}$ is defined only when $1+x-y^2\geq 0 \quad\Rightarrow\quad x\geq y^2-1$, so the domain of f is $\{(x,y)\mid x\geq y^2-1\}$, all those points on or to the right of the parabola $x=y^2-1$.The range of f is $[0,\infty)$.



9. (a) $f(2,-1,6)=e^{\sqrt{6-2^2-(-1)^2}}=e^{\sqrt{1}}=e$.

(b) $e^{\sqrt{z-x^2-y^2}}$ is defined when $z-x^2-y^2\geq 0 \quad\Rightarrow\quad z\geq x^2+y^2$. Thus the domain of f is $\{(x,y,z)\mid z\geq x^2+y^2\}$.

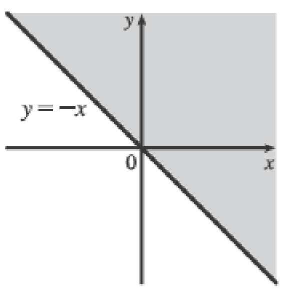
(c) Since $\sqrt{z-x^2-y^2}\geq 0$, we have $e^{\sqrt{z-x^2-y^2}}\geq 1$. Thus the range of f is $[1,\infty)$.

10. (a) $g(2,-2,4)=\ln(25-2^2-(-2)^2-4^2)=\ln 1=0$.

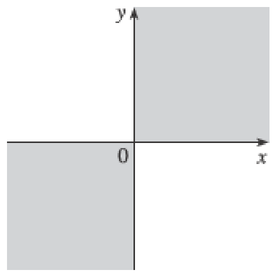
(b) For the logarithmic function to be defined, we need $25-x^2-y^2-z^2>0$. Thus the domain of g is $\{(x,y,z)\mid x^2+y^2+z^2<25\}$, the interior of the sphere $x^2+y^2+z^2=25$.

(c) Since $0<25-x^2-y^2-z^2\leq 25$ for (x,y,z) in the domain of g, $\ln(25-x^2-y^2-z^2)\leq \ln 25$. Thus the range of g is $(-\infty,\ln 25]$.

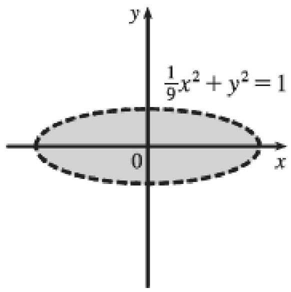
11. $\sqrt{x+y}$ is defined only when $x+y\geq 0$, or $y\geq -x$. So the domain of f is $\{(x,y)\mid y\geq -x\}$.



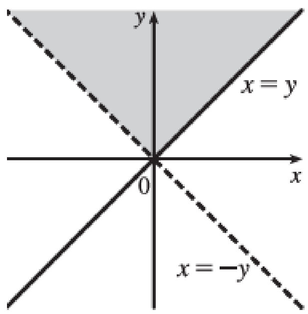
12. We need $xy \geq 0$, so $D = \{(x, y) \mid xy \geq 0\}$, the first and third quadrants.



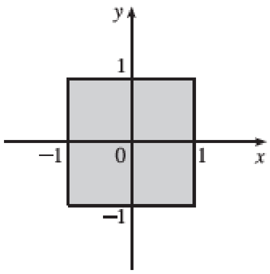
13. $\ln(9 - x^2 - 9y^2)$ is defined only when $9 - x^2 - 9y^2 > 0$, or $\frac{1}{9}x^2 + y^2 < 1$. So the domain of f is $\left\{(x, y) \mid \frac{1}{9}x^2 + y^2 < 1\right\}$, the interior of an ellipse.



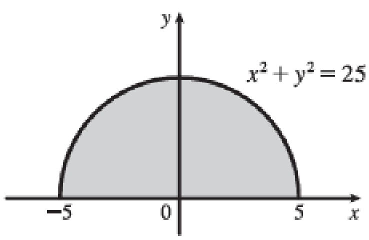
14. We need $y - x \geq 0$ or $y \geq x$ and $y + x > 0$ or $x > -y$. Thus $D = \{(x, y) \mid -y < x \leq y, y > 0\}$.



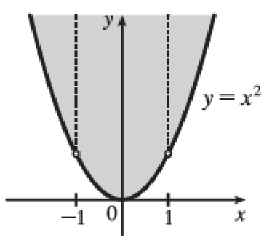
15. $\sqrt{1-x^2}$ is defined only when $1-x^2 \geq 0$, or $x^2 \leq 1$ $\Leftrightarrow$ $-1 \leq x \leq 1$, and $\sqrt{1-y^2}$ is defined only when $1-y^2 \geq 0$, or $y^2 \leq 1$ $\Leftrightarrow$ $-1 \leq y \leq 1$. Thus the domain of f is $\{(x,y) \mid -1 \leq x \leq 1,\ -1 \leq y \leq 1\}$.



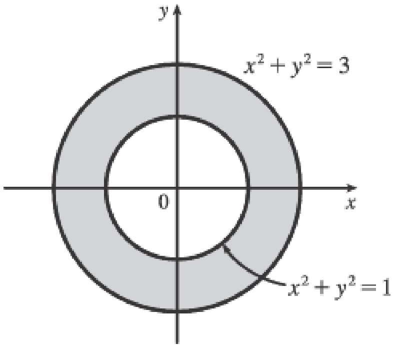
16. $\sqrt{y} + \sqrt{25-x^2-y^2}$ is defined only when $y \geq 0$ and $25-x^2-y^2 \geq 0$ $\Leftrightarrow$ $x^2+y^2 \leq 25$. So the domain of f is $\{(x,y) \mid x^2+y^2 \leq 25,\ y \geq 0\}$, a half disk of radius 5.



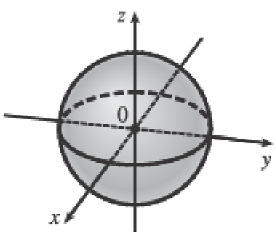
17. $\sqrt{y-x^2}$ is defined only when $y-x^2 \geq 0$, or $y \geq x^2$. In addition, f is not defined if $1-x^2=0$ $\Rightarrow$ $x = \pm 1$. Thus the domain of f is $\{(x,y) \mid y \geq x^2,\ x \neq \pm 1\}$.



18. $\arcsin(x^2 + y^2 - 2)$ is defined only when $-1 \le x^2 + y^2 - 2 \le 1 \iff 1 \le x^2 + y^2 \le 3$. Thus the domain of f is $\{(x, y) \mid 1 \le x^2 + y^2 \le 3\}$.



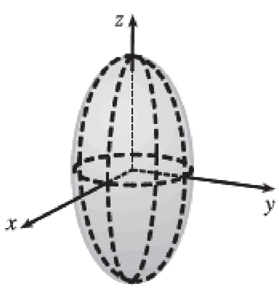
19. We need $1 - x^2 - y^2 - z^2 \ge 0$ or $x^2 + y^2 + z^2 \le 1$, so $D = \{(x, y, z) \mid x^2 + y^2 + z^2 \le 1\}$ (the points inside or on the sphere of radius 1, center the origin).



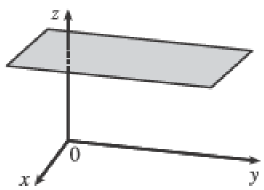
20. f is defined only when $16 - 4x^2 - 4y^2 - z^2 > 0 \Rightarrow$

$\dfrac{x^2}{4} + \dfrac{y^2}{4} + \dfrac{z^2}{16} < 1$. Thus,

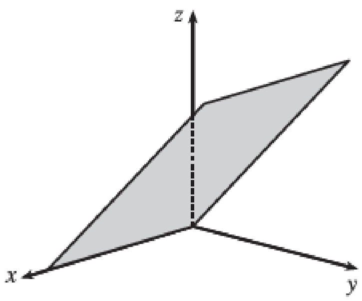
$D = \left\{(x, y, z) \,\middle|\, \dfrac{x^2}{4} + \dfrac{y^2}{4} + \dfrac{z^2}{16} < 1\right\}$, that is, the points inside the ellipsoid $\dfrac{x^2}{4} + \dfrac{y^2}{4} + \dfrac{z^2}{16} = 1$.



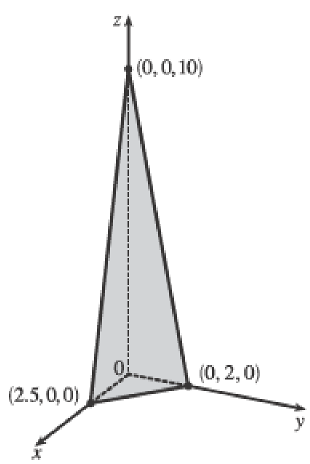
21. $z = 3$, a horizontal plane through the point $(0, 0, 3)$.



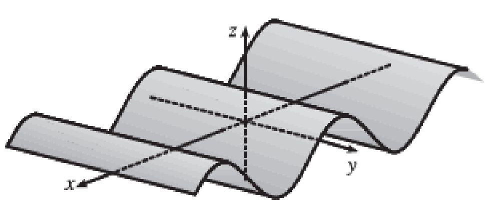
22. $z = y$, a plane which intersects the yz-plane in the line $z = y$, $x = 0$. The portion of this plane that lies in the first octant is shown.



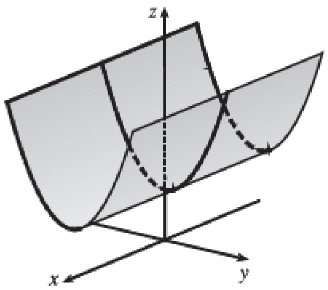
23. $z = 10 - 4x - 5y$ or $4x + 5y + z = 10$, a plane with intercepts 2.5, 2, and 10.



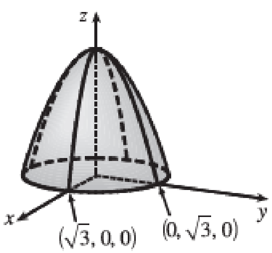
24. $z = \cos x$, a "wave."



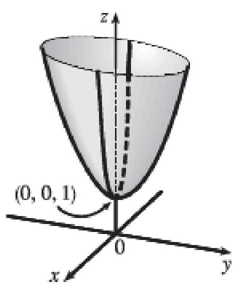
25. $z = y^2 + 1$, a parabolic cylinder



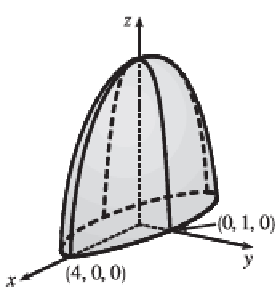
26. $z = 3 - x^2 - y^2$, a circular paraboloid with vertex at $(0, 0, 3)$.



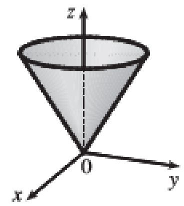
27. $z = 4x^2 + y^2 + 1$, an elliptic paraboloid with vertex at $(0, 0, 1)$.



28. $z = \sqrt{16 - x^2 - 16y^2}$ so $z \geq 0$ and $z^2 + x^2 + 16y^2 = 16$, the top half of an ellipsoid.



29. $z = \sqrt{x^2 + y^2}$ so $x^2 + y^2 = z^2$ and $z \geq 0$, the top half of a right circular cone.



30. All six graphs have different traces in the planes $x = 0$ and $y = 0$, so we investigate these for each function.

(a) $f(x, y) = |x| + |y|$. The trace in $x = 0$ is $z = |y|$, and in $y = 0$ is $z = |x|$, so it must be graph VI.

(b) $f(x, y) = |xy|$. The trace in $x = 0$ is $z = 0$, and in $y = 0$ is $z = 0$, so it must be graph V.

(c) $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$. The trace in $x = 0$ is $z = \dfrac{1}{1 + y^2}$, and in $y = 0$ is $z = \dfrac{1}{1 + x^2}$. In addition, we can see that f is close to 0 for large values of x and y, so this is graph I.

(d) $f(x, y) = (x^2 - y^2)^2$. The trace in $x = 0$ is $z = y^4$, and in $y = 0$ is $z = x^4$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x^2 - y^2)^2 \quad \Rightarrow \quad y = \pm x$, so it must be graph IV.

(e) $f(x, y) = (x - y)^2$. The trace in $x = 0$ is $z = y^2$, and in $y = 0$ is $z = x^2$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x - y)^2 \quad \Rightarrow \quad y = x$, so it must be graph II.

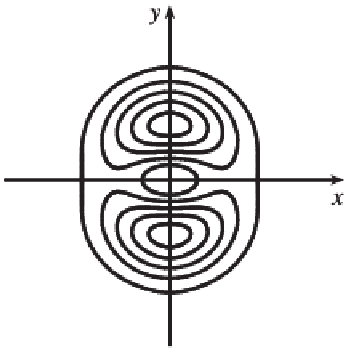
(f) $f(x, y) = \sin(|x| + |y|)$. The trace in $x = 0$ is $z = \sin|y|$, and in $y = 0$ is $z = \sin|x|$. In addition, notice that the oscillating nature of the graph is characteristic of trigonometric functions. So this is graph III.

31. The point $(-3, 3)$ lies between the level curves with z-values 50 and 60. Since the point is a little closer to the level curve with $z = 60$, we estimate that $f(-3, 3) \approx 56$. The point $(3, -2)$ appears to be just about halfway between the level curves with z-values 30 and 40, so we estimate $f(3, -2) \approx 35$. The graph rises as we approach the origin, gradually from above, steeply from below.

32. If we start at the origin and move along the x-axis, for example, the z-values of a cone centered at the origin increase at a constant rate, so we would expect its level curves to be equally spaced. A paraboloid with vertex the origin, on the other hand, has z-values which change slowly near the origin and more quickly as we move farther away. Thus, we would expect its level curves near the origin to be spaced more widely apart than those farther from the origin. Therefore contour map I must correspond to the paraboloid, and contour map II the cone.

33. Near A, the level curves are very close together, indicating that the terrain is quite steep. At B, the level curves are much farther apart, so we would expect the terrain to be much less steep than near A, perhaps almost flat.

34.



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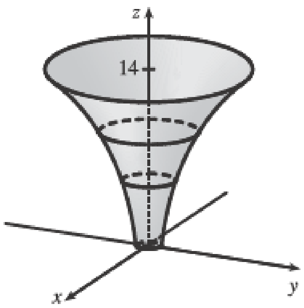

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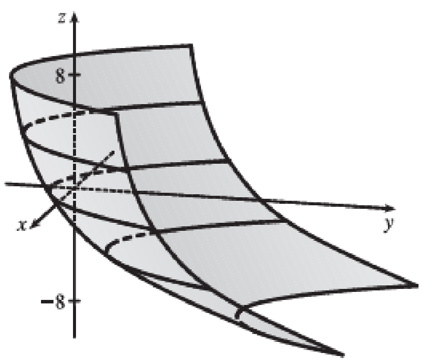

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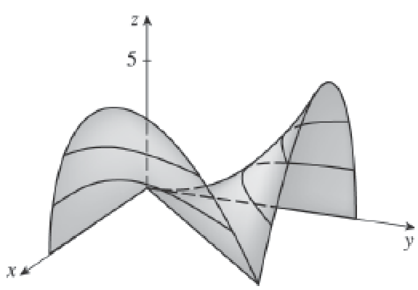

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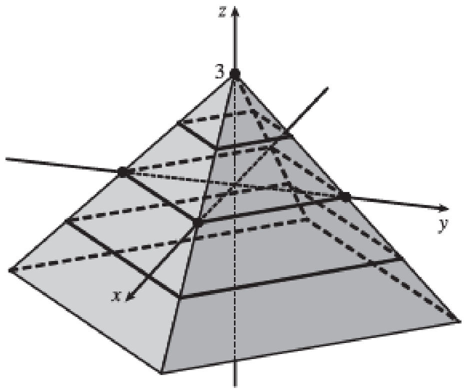
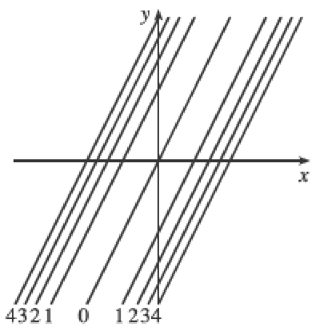
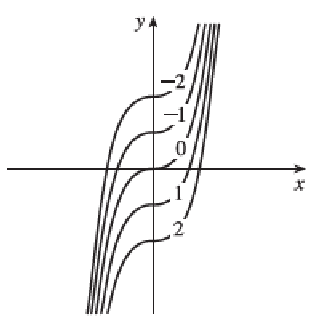


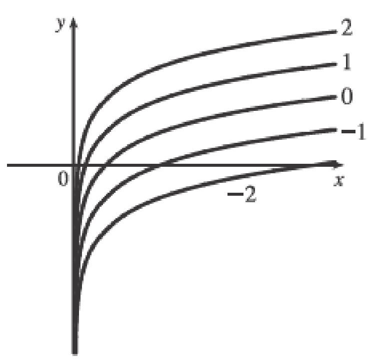
39. The level curves are $(y - 2x)^2 = k$ or $y = 2x \pm \sqrt{k}$, $k \geq 0$, a family of pairs of parallel lines.



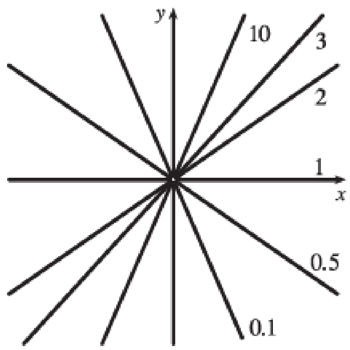
40. The level curves are $x^3 - y = k$ or $y = x^3 - k$, a family of cubic curves.



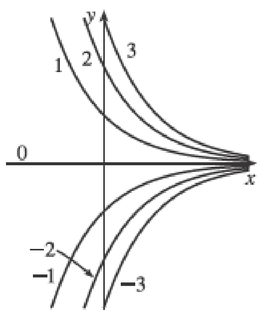
41. The level curves are $y - \ln x = k$ or $y = \ln x + k$.



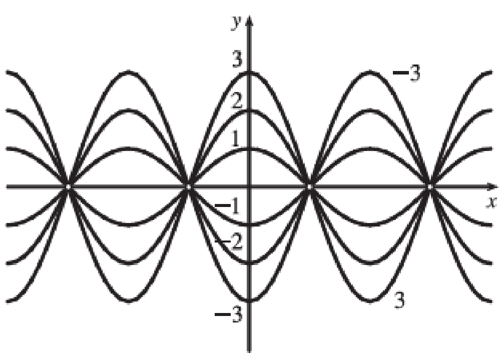
42. The level curves are $e^{y/x} = k$ or equivalently $y = x \ln k$ $(x \neq 0)$, a family of lines with slope $\ln k$ $(k > 0)$ without the origin.



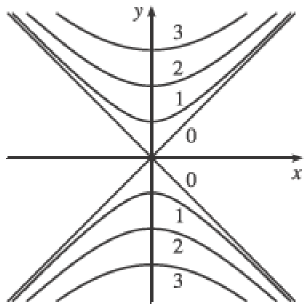
43. The level curves are $ye^x = k$ or $y = ke^{-x}$, a family of exponential curves.



44. $k = y \sec x$ or $y = k \cos x$, $x \neq \frac{\pi}{2} + n\pi$ [n an integer].

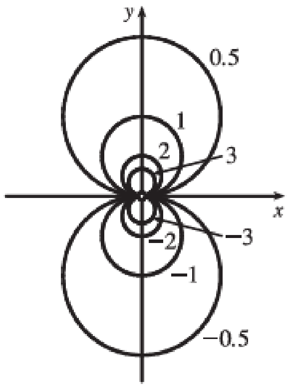


45. The level curves are $\sqrt{y^2 - x^2} = k$ or $y^2 - x^2 = k^2$, $k \geq 0$. When $k = 0$ the level curve is the pair of lines $y = \pm x$. For $k > 0$, the level curves are hyperbolas with axis the y-axis.

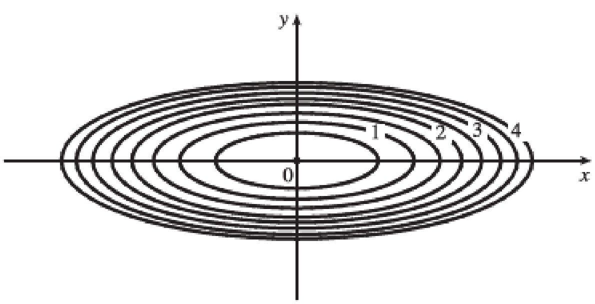


46. For $k \neq 0$ and $(x, y) \neq (0, 0)$, $k = \dfrac{y}{x^2 + y^2} \quad \Leftrightarrow$

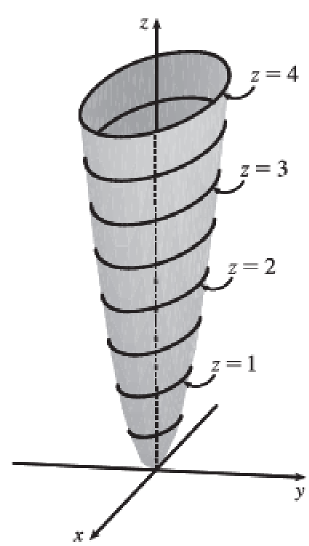
$x^2 + y^2 - \dfrac{y}{k} = 0 \quad \Leftrightarrow \quad x^2 + \left(y - \frac{1}{2k}\right)^2 = \dfrac{1}{4k^2}$, a family of circles with center $\left(0, \frac{1}{2k}\right)$ and radius $\frac{1}{2k}$ (without the origin). If $k = 0$, the level curve is the x-axis.

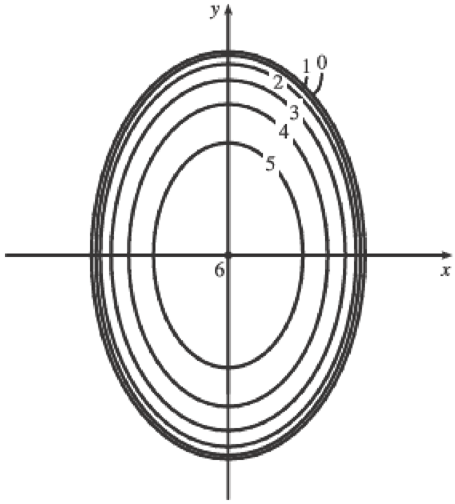


47. The contour map consists of the level curves $k = x^2 + 9y^2$, a family of ellipses with major axis the x-axis. (Or, if $k = 0$, the origin.) The graph of $f(x, y)$ is the surface $z = x^2 + 9y^2$, an elliptic paraboloid.

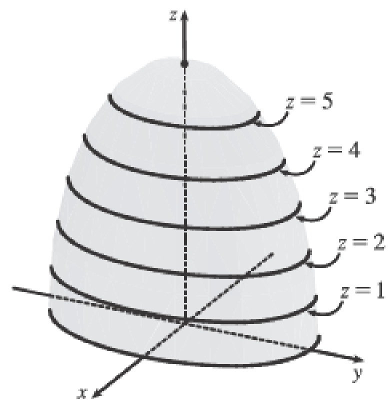


If we visualize lifting each ellipse $k = x^2 + 9y^2$ of the contour map to the plane $z = k$, we have horizontal traces that indicate the shape of the graph of f.



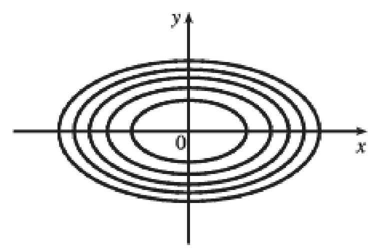
48. 

The contour map consists of the level curves $k = \sqrt{36 - 9x^2 - 4y^2} \Rightarrow 9x^2 + 4y^2 = 36 - k^2$, $k \geq 0$, a family of ellipses with major axis the y-axis. (Or, if $k = 6$, the origin.)



The graph of $f(x, y)$ is the surface $z = \sqrt{36 - 9x^2 - 4y^2}$, or equivalently the upper half of the ellipsoid $9x^2 + 4y^2 + z^2 = 36$. If we visualize lifting each ellipse $k = \sqrt{36 - 9x^2 - 4y^2}$ of the contour map to the plane $z = k$, we have horizontal traces that indicate the shape of the graph of f.

49. The isothermals are given by $k = 100/(1 + x^2 + 2y^2)$ or $x^2 + 2y^2 = (100 - k)/k$ $[0 < k \leq 100]$, a family of ellipses.



50. The equipotential curves are $k = \dfrac{c}{\sqrt{r^2 - x^2 - y^2}}$ or $x^2 + y^2 = r^2 - \left(\frac{c}{k}\right)^2$, a family of circles ($k \geq c/r$).

Note: As $k \to \infty$, the radius of the circle approaches r.