1. $\int_0^5 12x^2y^3\,dx = \left[12\,\frac{x^3}{3}\,y^3\right]_{x=0}^{x=5} = 4x^3y^3\big]_{x=0}^{x=5} = 4(5)^3\,y^3 - 4(0)^3\,y^3 = 500y^3,$

$\int_0^1 12x^2y^3\,dy = \left[12x^2\,\frac{y^4}{4}\right]_{y=0}^{y=1} = 3x^2y^4\big]_{y=0}^{y=1} = 3x^2(1)^4 - 3x^2(0)^4 = 3x^2$

2. $\int_0^5 (y + xe^y)\,dx = \left[xy + \frac{x^2}{2}e^y\right]_{x=0}^{x=5} = \left(5y + \frac{25}{2}e^y\right) - (0 + 0) = 5y + \frac{25}{2}e^y,$

$\int_0^1 (y + xe^y)\,dy = \left[\frac{y^2}{2} + xe^y\right]_{y=0}^{y=1} = \left(\frac{1}{2} + xe^1\right) - (0 + xe^0) = \frac{1}{2} + ex - x$

3. $\int_1^3 \int_0^1 (1 + 4xy)\,dx\,dy = \int_1^3 \left[x + 2x^2y\right]_{x=0}^{x=1}\,dy = \int_1^3 (1 + 2y)\,dy = \left[y + y^2\right]_1^3 = (3 + 9) - (1 + 1) = 10$

4. $\int_0^1 \int_1^2 (4x^3 - 9x^2y^2)\,dy\,dx = \int_0^1 \left[4x^3y - 3x^2y^3\right]_{y=1}^{y=2}\,dx = \int_0^1 \left[(8x^3 - 24x^2) - (4x^3 - 3x^2)\right]\,dx$

$= \int_0^1 (4x^3 - 21x^2)\,dx = \left[x^4 - 7x^3\right]_0^1 = (1 - 7) - (0 - 0) = -6$

5. $\int_0^2 \int_0^{\pi/2} x\sin y\,dy\,dx = \int_0^2 x\,dx \int_0^{\pi/2} \sin y\,dy$ [as in Example 5] $= \left[\frac{x^2}{2}\right]_0^2 \left[-\cos y\right]_0^{\pi/2} = (2 - 0)(0 + 1) = 2$

6. $\int_{\pi/6}^{\pi/2} \int_{-1}^{5} \cos y\,dx\,dy = \int_{-1}^{5} dx \int_{\pi/6}^{\pi/2} \cos y\,dy$ [by Equation 5]

$= \left[x\right]_{-1}^{5}\left[\sin y\right]_{\pi/6}^{\pi/2} = [5 - (-1)](\sin\frac{\pi}{2} - \sin\frac{\pi}{6}) = 6(1 - \frac{1}{2}) = 3$

7. $\int_0^2 \int_0^1 (2x + y)^8\,dx\,dy = \int_0^2 \left[\frac{1}{2}\frac{(2x + y)^9}{9}\right]_{x=0}^{x=1} dy$ [substitute $u = 2x + y \Rightarrow dx = \frac{1}{2}\,du$]

$= \frac{1}{18}\int_0^2 \left[(2 + y)^9 - (0 + y)^9\right] dy = \frac{1}{18}\left[\frac{(2 + y)^{10}}{10} - \frac{y^{10}}{10}\right]_0^2$

$= \frac{1}{180}\left[(4^{10} - 2^{10}) - (2^{10} - 0^{10})\right] = \frac{1{,}046{,}528}{180} = \frac{261{,}632}{45}$

8. $\int_0^1 \int_1^2 \frac{xe^x}{y}\,dy\,dx = \int_0^1 xe^x\,dx \int_1^2 \frac{1}{y}\,dy$ [as in Example 5] $= \left[xe^x - e^x\right]_0^1 \left[\ln|y|\right]_1^2$ [by integrating by parts]

$= [(e - e) - (0 - 1)](\ln 2 - 0) = \ln 2$

9. $\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dy\,dx = \int_1^4 \left[x\ln|y| + \frac{1}{x}\cdot\frac{1}{2}y^2\right]_{y=1}^{y=2} dx = \int_1^4 \left(x\ln 2 + \frac{3}{2x}\right) dx = \left[\frac{1}{2}x^2\ln 2 + \frac{3}{2}\ln|x|\right]_1^4$

$= 8\ln 2 + \frac{3}{2}\ln 4 - \frac{1}{2}\ln 2 = \frac{15}{2}\ln 2 + 3\ln 4^{1/2} = \frac{21}{2}\ln 2$

10. $\int_0^1 \int_0^3 e^{x+3y}\,dx\,dy = \int_0^1 \int_0^3 e^x e^{3y}\,dx\,dy = \int_0^3 e^x\,dx \int_0^1 e^{3y}\,dy = \left[e^x\right]_0^3 \left[\frac{1}{3}e^{3y}\right]_0^1$

$= \left(e^3 - e^0\right)\cdot\frac{1}{3}\left(e^3 - e^0\right) = \frac{1}{3}(e^3 - 1)^2 or \frac{1}{3}(e^6 - 2e^3 + 1)$

11. $\int_0^1 \int_0^1 (u-v)^5 du\, dv = \int_0^1 \left[\frac{1}{6}(u-v)^6\right]_{u=0}^{u=1} dv = \frac{1}{6}\int_0^1 \left[(1-v)^6 - (0-v)^6\right] dv$
$= \frac{1}{6}\int_0^1 \left[(1-v)^6 - v^6\right] dv = \frac{1}{6}\left[-\frac{1}{7}(1-v)^7 - \frac{1}{7}v^7\right]_0^1$
$= -\frac{1}{42}\left[(0+1)-(1+0)\right] = 0$

12. $\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dy\,dx = \int_0^1 x\left[\frac{1}{3}(x^2+y^2)^{3/2}\right]_{y=0}^{y=1} dx = \frac{1}{3}\int_0^1 x\left[(x^2+1)^{3/2} - x^3\right] dx = \frac{1}{3}\int_0^1 [x(x^2+1)^{3/2} - x^4]dx$
$= \frac{1}{3}\left[\frac{1}{5}(x^2+1)^{5/2} - \frac{1}{5}x^5\right]_0^1 = \frac{1}{15}\left[2^{5/2} - 1 - 1 + 0\right] = \frac{2}{15}(2\sqrt{2}-1)$

13. $\int_0^2\int_0^\pi r\sin^2\theta\,d\theta\,dr = \int_0^2 r\,dr \int_0^\pi \sin^2\theta\,d\theta$ [as in Example 5] $= \int_0^2 r\,dr\int_0^\pi \frac{1}{2}(1-\cos 2\theta)\,d\theta$
$= \left[\frac{1}{2}r^2\right]_0^2 \cdot \frac{1}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^\pi = (2-0)\cdot\frac{1}{2}\left[\left(\pi - \frac{1}{2}\sin 2\pi\right) - \left(0 - \frac{1}{2}\sin 0\right)\right]$
$= 2\cdot\frac{1}{2}[(\pi-0)-(0-0)] = \pi$

14. $\int_0^1\int_0^1 \sqrt{s+t}\,ds\,dt = \int_0^1 \left[\frac{2}{3}(s+t)^{3/2}\right]_{s=0}^{s=1} dt = \frac{2}{3}\int_0^1 [(1+t)^{3/2} - t^{3/2}]\,dt = \frac{2}{3}\left[\frac{2}{5}(1+t)^{5/2} - \frac{2}{5}t^{5/2}\right]_0^1$
$= \frac{4}{15}[(2^{5/2}-1)-(1-0)] = \frac{4}{15}\left(2^{5/2}-2\right)$ or $\frac{8}{15}(2\sqrt{2}-1)$

15. $\iint_R (6x^2y^3 - 5y^4)\,dA = \int_0^3\int_0^1 (6x^2y^3 - 5y^4)\,dy\,dx = \int_0^3 \left[\frac{3}{2}x^2y^4 - y^5\right]_{y=0}^{y=1} dx = \int_0^3 \left(\frac{3}{2}x^2 - 1\right) dx$
$= \left[\frac{1}{2}x^3 - x\right]_0^3 = \frac{27}{2} - 3 = \frac{21}{2}$

16. $\iint_R \cos(x+2y)\,dA = \int_0^\pi\int_0^{\pi/2}\cos(x+2y)\,dy\,dx = \int_0^\pi \left[\frac{1}{2}\sin(x+2y)\right]_{y=0}^{y=\pi/2} dx = \frac{1}{2}\int_0^\pi (\sin(x+\pi) - \sin x)\,dx$
$= \frac{1}{2}\left[-\cos(x+\pi) + \cos x\right]_0^\pi = \frac{1}{2}\left[-\cos 2\pi + \cos\pi - (-\cos\pi + \cos 0)\right]$
$= \frac{1}{2}(-1-1-(1+1)) = -2$

17. $\iint_R \frac{xy^2}{x^2+1}\,dA = \int_0^1\int_{-3}^3 \frac{xy^2}{x^2+1}\,dy\,dx = \int_0^1 \frac{x}{x^2+1}\,dx\int_{-3}^3 y^2\,dy = \left[\frac{1}{2}\ln(x^2+1)\right]_0^1 \left[\frac{1}{3}y^3\right]_{-3}^3$
$= \frac{1}{2}(\ln 2 - \ln 1)\cdot\frac{1}{3}(27+27) = 9\ln 2$

18. $\iint_R \frac{1+x^2}{1+y^2}\,dA = \int_0^1\int_0^1 \frac{1+x^2}{1+y^2}\,dy\,dx = \int_0^1 (1+x^2)\,dx\int_0^1 \frac{1}{1+y^2}\,dy = \left[x + \frac{1}{3}x^3\right]_0^1 \left[\tan^{-1} y\right]_0^1$
$= \left(1+\frac{1}{3}-0\right)\left(\frac{\pi}{4}-0\right) = \frac{\pi}{3}$

19. $\int_0^{\pi/6}\int_0^{\pi/3} x\sin(x+y)\,dy\,dx$
$= \int_0^{\pi/6}\left[-x\cos(x+y)\right]_{y=0}^{y=\pi/3} dx = \int_0^{\pi/6}\left[x\cos x - x\cos\left(x+\frac{\pi}{3}\right)\right] dx$
$= x\left[\sin x - \sin\left(x+\frac{\pi}{3}\right)\right]_0^{\pi/6} - \int_0^{\pi/6}\left[\sin x - \sin\left(x+\frac{\pi}{3}\right)\right] dx$ [by integrating by parts separately for each term]
$= \frac{\pi}{6}\left[\frac{1}{2}-1\right] - \left[-\cos x + \cos\left(x+\frac{\pi}{3}\right)\right]_0^{\pi/6} = -\frac{\pi}{12} - \left[-\frac{\sqrt{3}}{2} + 0 - \left(-1+\frac{1}{2}\right)\right] = \frac{\sqrt{3}-1}{2} - \frac{\pi}{12}$

20. $\displaystyle\iint_R \frac{x}{1+xy}\,dA = \int_0^1 \int_0^1 \frac{x}{1+xy}\,dy\,dx = \int_0^1 \left[\ln(1+xy)\right]_{y=0}^{y=1}\,dx = \int_0^1 \left[\ln(1+x) - \ln 1\right]dx$

$\displaystyle = \int_0^1 \ln(1+x)\,dx = \left[(1+x)\ln(1+x) - x\right]_0^1$ [by integrating by parts]

$= (2\ln 2 - 1) - (\ln 1 - 0) = 2\ln 2 - 1$

21. $\displaystyle\iint_R xye^{x^2y}\,dA = \int_0^2 \int_0^1 xye^{x^2y}\,dx\,dy = \int_0^2 \left[\tfrac{1}{2}e^{x^2y}\right]_{x=0}^{x=1} dy = \tfrac{1}{2}\int_0^2 (e^y - 1)\,dy = \tfrac{1}{2}\left[e^y - y\right]_0^2$

$= \tfrac{1}{2}[(e^2 - 2) - (1 - 0)] = \tfrac{1}{2}(e^2 - 3)$

22. $\displaystyle\int_0^1 \int_1^2 \frac{x}{x^2+y^2}\,dx\,dy = \int_0^1 \left[\tfrac{1}{2}\ln(x^2+y^2)\right]_{x=1}^{x=2} dy = \tfrac{1}{2}\int_0^1 \left[\ln(4+y^2) - \ln(1+y^2)\right]dy$

To evaluate the first term, we integrate by parts with $u = \ln(4+y^2) \;\Rightarrow\; du = \dfrac{2y}{4+y^2}\,dy$ and

$dv = dy \;\Rightarrow\; v = y$. Then

$$\int \ln(4+y^2)\,dy = y\ln(4+y^2) - \int \frac{2y^2}{4+y^2}\,dy = y\ln(4+y^2) - \int \left(2 - \frac{8}{4+y^2}\right)dy$$

$$= y\ln(4+y^2) - 2y + 8\cdot\tfrac{1}{2}\tan^{-1}\left(\tfrac{y}{2}\right) = y\ln(4+y^2) - 2y + 4\tan^{-1}\left(\tfrac{y}{2}\right)$$

Similarly, $\int \ln(1+y^2)\,dy = y\ln(1+y^2) - 2y + 2\tan^{-1} y$. Thus,

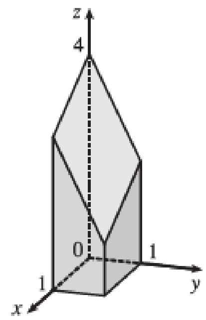
$$\int_0^1 \int_1^2 \frac{x}{x^2+y^2}\,dx\,dy = \tfrac{1}{2}\int_0^1 \left[\ln(4+y^2) - \ln(1+y^2)\right]dy$$

$$= \tfrac{1}{2}\left[y\ln(4+y^2) - 2y + 4\tan^{-1}\left(\tfrac{y}{2}\right) - y\ln(1+y^2) + 2y - 2\tan^{-1} y\right]_0^1$$

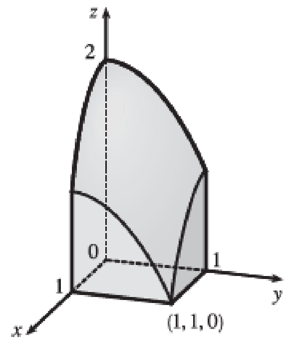
$$= \tfrac{1}{2}\left[\left(\ln 5 + 4\tan^{-1}\left(\tfrac{1}{2}\right) - \ln 2 - 2\tan^{-1} 1\right) - 0\right]$$

$$= \tfrac{1}{2}\left[\ln 5 - \ln 2 + 4\tan^{-1}\left(\tfrac{1}{2}\right) - 2\left(\tfrac{\pi}{4}\right)\right] = \tfrac{1}{2}\ln\tfrac{5}{2} + 2\tan^{-1}\left(\tfrac{1}{2}\right) - \tfrac{\pi}{4}$$

23. $z = f(x,y) = 4 - x - 2y \ge 0$ for $0 \le x \le 1$ and $0 \le y \le 1$. So the solid is the region in the first octant which lies below the plane $z = 4 - x - 2y$ and above $[0,1]\times[0,1]$.



24. $z = 2 - x^2 - y^2 \geq 0$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$. So the solid is the region in the first octant which lies below the circular paraboloid $z = 2 - x^2 - y^2$ and above $[0, 1] \times [0, 1]$.



25. $V = \iint_R (12 - 3x - 2y)\, dA = \int_{-2}^{3} \int_0^1 (12 - 3x - 2y)\, dx\, dy = \int_{-2}^{3} \left[12x - \frac{3}{2}x^2 - 2xy\right]_{x=0}^{x=1} dy$

$= \int_{-2}^{3} \left(\frac{21}{2} - 2y\right) dy = \left[\frac{21}{2}y - y^2\right]_{-2}^{3} = \frac{95}{2}$

26. $V = \iint_R (4 + x^2 - y^2)\, dA = \int_{-1}^{1} \int_0^2 (4 + x^2 - y^2)\, dy\, dx = \int_{-1}^{1} \left[4y + x^2 y - \frac{1}{3}y^3\right]_{y=0}^{y=2} dx$

$= \int_{-1}^{1} \left(2x^2 + \frac{16}{3}\right) dx = \left[\frac{2}{3}x^3 + \frac{16}{3}x\right]_{-1}^{1} = \frac{2}{3} + \frac{16}{3} + \frac{2}{3} + \frac{16}{3} = 12$

27. $V = \int_{-2}^{2} \int_{-1}^{1} \left(1 - \frac{1}{4}x^2 - \frac{1}{9}y^2\right) dx\, dy = 4 \int_0^2 \int_0^1 \left(1 - \frac{1}{4}x^2 - \frac{1}{9}y^2\right) dx\, dy$

$= 4 \int_0^2 \left[x - \frac{1}{12}x^3 - \frac{1}{9}y^2 x\right]_{x=0}^{x=1} dy = 4 \int_0^2 \left(\frac{11}{12} - \frac{1}{9}y^2\right) dy = 4\left[\frac{11}{12}y - \frac{1}{27}y^3\right]_0^2 = 4 \cdot \frac{83}{54} = \frac{166}{27}$

28. $V = \int_{-1}^{1} \int_0^{\pi} (1 + e^x \sin y)\, dy\, dx = \int_{-1}^{1} \left[y - e^x \cos y\right]_{y=0}^{y=\pi} dx = \int_{-1}^{1} (\pi + e^x - 0 + e^x)\, dx$

$= \int_{-1}^{1} (\pi + 2e^x)\, dx = \left[\pi x + 2e^x\right]_{-1}^{1} = 2\pi + 2e - \frac{2}{e}$

29. Here we need the volume of the solid lying under the surface $z = x \sec^2 y$ and above the rectangle $R = [0, 2] \times [0, \pi/4]$ in the xy-plane.

$$V = \int_0^2 \int_0^{\pi/4} x \sec^2 y\, dy\, dx = \int_0^2 x\, dx \int_0^{\pi/4} \sec^2 y\, dy = \left[\frac{1}{2}x^2\right]_0^2 \left[\tan y\right]_0^{\pi/4}$$

$$= (2 - 0)(\tan \tfrac{\pi}{4} - \tan 0) = 2(1 - 0) = 2$$

30. The cylinder intersects the xy-plane along the line $x = 4$, so in the first octant, the solid lies below the surface $z = 16 - x^2$ and above the rectangle $R = [0, 4] \times [0, 5]$ in the xy-plane.

$$V = \int_0^5 \int_0^4 (16 - x^2)\, dx\, dy = \int_0^4 (16 - x^2)\, dx \int_0^5 dy = \left[16x - \frac{1}{3}x^3\right]_0^4 \left[y\right]_0^5 = (64 - \tfrac{64}{3} - 0)(5 - 0) = \tfrac{640}{3}$$

31. The solid lies below the surface $z = 2 + x^2 + (y-2)^2$ and above the plane $z = 1$ for $-1 \le x \le 1$, $0 \le y \le 4$. The volume of the solid is the difference in volumes between the solid that lies under $z = 2 + x^2 + (y-2)^2$ over the rectangle $R = [-1, 1] \times [0, 4]$ and the solid that lies under $z = 1$ over R.

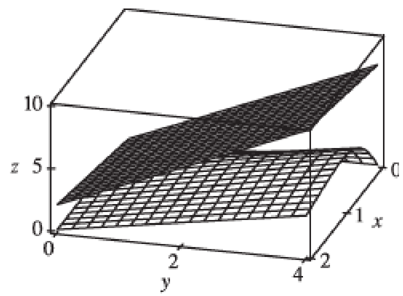
$$V = \int_0^4 \int_{-1}^1 [2 + x^2 + (y-2)^2]\, dx\, dy - \int_0^4 \int_{-1}^1 (1)\, dx\, dy = \int_0^4 \left[2x + \tfrac{1}{3}x^3 + x(y-2)^2\right]_{x=-1}^{x=1} dy - \int_{-1}^1 dx \int_0^4 dy$$

$$= \int_0^4 \left[(2 + \tfrac{1}{3} + (y-2)^2) - (-2 - \tfrac{1}{3} - (y-2)^2)\right] dy - [x]_{-1}^1\, [y]_0^4$$

$$= \int_0^4 \left[\tfrac{14}{3} + 2(y-2)^2\right] dy - [1 - (-1)][4 - 0] = \left[\tfrac{14}{3}y + \tfrac{2}{3}(y-2)^3\right]_0^4 - (2)(4)$$

$$= \left[(\tfrac{56}{3} + \tfrac{16}{3}) - (0 - \tfrac{16}{3})\right] - 8 = \tfrac{88}{3} - 8 = \tfrac{64}{3}$$

32.



The solid lies below the plane $z = x + 2y$ and above the surface $z = \dfrac{2xy}{x^2+1}$ for $0 \le x \le 2$, $0 \le y \le 4$. The volume of the solid is the difference in volumes between the solid that lies under $z = x + 2y$ over the rectangle $R = [0, 2] \times [0, 4]$ and the solid that lies under $z = \dfrac{2xy}{x^2+1}$ over R.

$$V = \int_0^2 \int_0^4 (x + 2y)\,dy\, dx - \int_0^2 \int_0^4 \frac{2xy}{x^2+1}\, dy\, dx = \int_0^2 \left[xy + y^2\right]_{y=0}^{y=4} dx - \int_0^2 \frac{2x}{x^2+1}\, dx \int_0^4 y\, dy$$

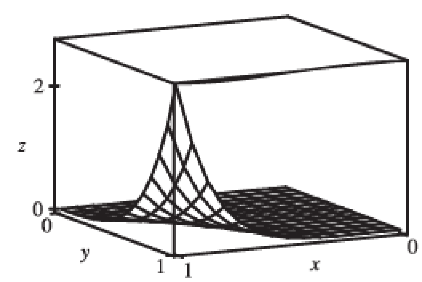
$$= \int_0^2 [(4x + 16) - (0 + 0)]\, dx - \left[\ln\left|x^2 + 1\right|\right]_0^2 \left[\tfrac{1}{2}y^2\right]_0^4$$

$$= \left[2x^2 + 16x\right]_0^2 - (\ln 5 - \ln 1)(8 - 0) = (8 + 32 - 0) - 8 \ln 5 = 40 - 8 \ln 5$$

33. In Maple, we can calculate the integral by defining the integrand as `f` and then using the command `int(int(f,x=0..1),y=0..1);`. In Mathematica, we can use the command

```
Integrate[Integrate[f,{x,0,1}],{y,0,1}];
```

We find that $\iint_R x^5 y^3 e^{xy}\, dA = 21e - 57 \approx 0.0839$. We can use `plot3d` (in Maple) or `Plot3D` (in Mathematica) to graph the function.



34. In Maple, we can calculate the integral by defining

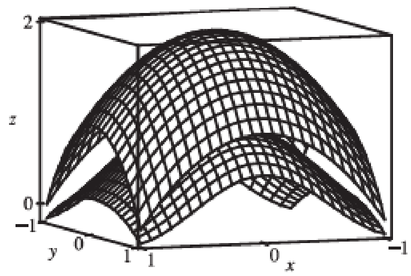
`f:=exp(-x^2)*cos(x^2+y^2);` and `g:=2-x^2-y^2;`

and then [since $2 - x^2 - y^2 > e^{-x^2}\cos(x^2 + y^2)$ for $-1 \le x \le 1, -1 \le y \le 1$] using the command

`evalf(int(int(g-f,x=-1..1),y=-1..1),5);`.

In Mathematica, we can use the command

`N[Integrate[Integrate[f,{x,0,1}],{y,0,1}],5]`.



In each of these commands, the `5` indicates that we want only five significant digits; this speeds up the calculation considerably. We find that $\iint_R \left[(2 - x^2 - y^2) - \left(e^{-x^2}\cos(x^2 + y^2)\right)\right] dA \approx 3.0271$. We can use the `plot3d` command (in Maple) or `Plot3d` (in Mathematica) to graph both functions on the same screen.

35. R is the rectangle $[-1, 1] \times [0, 5]$. Thus, $A(R) = 2 \cdot 5 = 10$ and

$$f_{\text{ave}} = \frac{1}{A(R)} \iint_R f(x, y)\, dA = \tfrac{1}{10} \int_0^5 \int_{-1}^1 x^2 y\, dx\, dy = \tfrac{1}{10} \int_0^5 \left[\tfrac{1}{3}x^3 y\right]_{x=-1}^{x=1} dy = \tfrac{1}{10} \int_0^5 \tfrac{2}{3} y\, dy = \tfrac{1}{10}\left[\tfrac{1}{3}y^2\right]_0^5 = \tfrac{5}{6}.$$

36. $A(R) = 4 \cdot 1 = 4$, so

$$f_{\text{ave}} = \frac{1}{A(R)} \iint_R f(x, y)\, dA = \tfrac{1}{4} \int_0^4 \int_0^1 e^y \sqrt{x + e^y}\, dy\, dx = \tfrac{1}{4} \int_0^4 \left[\tfrac{2}{3}(x + e^y)^{3/2}\right]_{y=0}^{y=1} dx$$

$$= \tfrac{1}{4} \cdot \tfrac{2}{3} \int_0^4 [(x + e)^{3/2} - (x + 1)^{3/2}]\, dx = \tfrac{1}{6}\left[\tfrac{2}{5}(x + e)^{5/2} - \tfrac{2}{5}(x + 1)^{5/2}\right]_0^4$$

$$= \tfrac{1}{6} \cdot \tfrac{2}{5}[(4 + e)^{5/2} - 5^{5/2} - e^{5/2} + 1] = \tfrac{1}{15}[(4 + e)^{5/2} - e^{5/2} - 5^{5/2} + 1] \approx 3.327$$

37. Let $f(x, y) = \dfrac{x - y}{(x + y)^3}$. Then a CAS gives $\int_0^1 \int_0^1 f(x, y)\, dy\, dx = \frac{1}{2}$ and $\int_0^1 \int_0^1 f(x, y)\, dx\, dy = -\frac{1}{2}$.

To explain the seeming violation of Fubini's Theorem, note that f has an infinite discontinuity at $(0, 0)$ and thus does not satisfy the conditions of Fubini's Theorem. In fact, both iterated integrals involve improper integrals which diverge at their lower limits of integration.

38. (a) Loosely speaking, Fubini's Theorem says that the order of integration of a function of two variables does not affect the value of the double integral, while Clairaut's Theorem says that the order of differentiation of such a function does not affect the value of the second-order derivative. Also, both theorems require continuity (though Fubini's allows a finite number of smooth curves to contain discontinuities).

(b) To find g_{xy}, we first hold y constant and use the single-variable Fundamental Theorem of Calculus, Part 1:

$g_x = \dfrac{d}{dx} g(x, y) = \dfrac{d}{dx} \displaystyle\int_a^x \left(\int_c^y f(s, t)\, dt\right) ds = \int_c^y f(x, t)\, dt$. Now we use the Fundamental Theorem again:

$$g_{xy} = \frac{d}{dy} \int_c^y f(x, t)\, dt = f(x, y).$$

To find g_{yx}, we first use Fubini's Theorem to find that $\int_a^x \int_c^y f(s, t)\, dt\, ds = \int_c^y \int_a^x f(s, t)\, dt\, ds$, and then use the Fundamental Theorem twice, as above, to get $g_{yx} = f(x, y)$. So $g_{xy} = g_{yx} = f(x, y)$.