1. $\int_0^4 \int_0^{\sqrt{y}} xy^2\,dx\,dy = \int_0^4 \left[\frac{1}{2}x^2y^2\right]_{x=0}^{x=\sqrt{y}} dy = \int_0^4 \frac{1}{2}y^2[(\sqrt{y}\,)^2 - 0^2]dy = \frac{1}{2}\int_0^4 y^3\,dy = \frac{1}{2}\left[\frac{1}{4}y^4\right]_0^4 = \frac{1}{2}(64-0) = 32$

2. $\int_0^1 \int_{2x}^2 (x-y)dy\,dx = \int_0^1 \left[xy - \frac{1}{2}y^2\right]_{y=2x}^{y=2} dx = \int_0^1 \left[x(2) - \frac{1}{2}(2)^2 - x(2x) + \frac{1}{2}(2x)^2\right] dx$

$= \int_0^1 (2x-2)dx = \left[x^2 - 2x\right]_0^1 = 1 - 2 - 0 + 0 = -1$

3. $\int_0^1 \int_{x^2}^x (1+2y)dy\,dx = \int_0^1 \left[y + y^2\right]_{y=x^2}^{y=x} dx = \int_0^1 \left[x + x^2 - x^2 - (x^2)^2\right] dx$

$= \int_0^1 (x - x^4)dx = \left[\frac{1}{2}x^2 - \frac{1}{5}x^5\right]_0^1 = \frac{1}{2} - \frac{1}{5} - 0 + 0 = \frac{3}{10}$

4. $\int_0^2 \int_y^{2y} xy\,dx\,dy = \int_0^2 \left[\frac{1}{2}x^2y\right]_{x=y}^{x=2y} dy = \int_0^2 \frac{1}{2}y(4y^2 - y^2)\,dy = \frac{1}{2}\int_0^2 3y^3\,dy = \frac{3}{2}\left[\frac{1}{4}y^4\right]_0^2 = \frac{3}{2}(4-0) = 6$

5. $\int_0^{\pi/2} \int_0^{\cos\theta} e^{\sin\theta} dr\,d\theta = \int_0^{\pi/2} \left[re^{\sin\theta}\right]_{r=0}^{r=\cos\theta} d\theta = \int_0^{\pi/2} (\cos\theta)\,e^{\sin\theta} d\theta = \left. e^{\sin\theta}\right]_0^{\pi/2} = e^{\sin(\pi/2)} - e^0 = e - 1$

6. $\int_0^1 \int_0^v \sqrt{1-v^2}\,du\,dv = \int_0^1 \left[u\sqrt{1-v^2}\,\right]_{u=0}^{u=v} dv = \int_0^1 v\sqrt{1-v^2}\,dv = \left. -\frac{1}{3}(1-v^2)^{3/2}\right]_0^1 = -\frac{1}{3}(0-1) = \frac{1}{3}$

7. $\iint_D y^2\,dA = \int_{-1}^1 \int_{-y-2}^y y^2\,dx\,dy = \int_{-1}^1 \left[xy^2\right]_{x=-y-2}^{x=y} dy = \int_{-1}^1 y^2\left[y - (-y-2)\right] dy$

$= \int_{-1}^1 (2y^3 + 2y^2)dy = \left[\frac{1}{2}y^4 + \frac{2}{3}y^3\right]_{-1}^1 = \frac{1}{2} + \frac{2}{3} - \frac{1}{2} + \frac{2}{3} = \frac{4}{3}$

8. $\displaystyle\iint_D \frac{y}{x^5+1}\,dA = \int_0^1 \int_0^{x^2} \frac{y}{x^5+1}\,dy\,dx = \int_0^1 \frac{1}{x^5+1}\left[\frac{y^2}{2}\right]_{y=0}^{y=x^2} dx = \frac{1}{2}\int_0^1 \frac{x^4}{x^5+1}\,dx = \frac{1}{2}\left[\frac{1}{5}\ln\left|x^5+1\right|\right]_0^1$

$= \frac{1}{10}(\ln 2 - \ln 1) = \frac{1}{10}\ln 2$

9. $\iint_D x\,dA = \int_0^\pi \int_0^{\sin x} x\,dy\,dx = \int_0^\pi \left[xy\right]_{y=0}^{y=\sin x} dx = \int_0^\pi x\sin x\,dx$ $\left[\begin{array}{c}\text{integrate by parts}\\ \text{with } u = x,\, dv = \sin x\,dx\end{array}\right]$

$= \left[-x\cos x + \sin x\right]_0^\pi = -\pi\cos\pi + \sin\pi + 0 - \sin 0 = \pi$

10. $\iint_D x^3\,dA = \int_1^e \int_0^{\ln x} x^3\,dy\,dx = \int_1^e \left[x^3y\right]_{y=0}^{y=\ln x} dx = \int_1^e x^3\ln x\,dx$ $\left[\begin{array}{c}\text{integrate by parts}\\ \text{with } u = \ln x,\, dv = x^3\,dx\end{array}\right]$

$= \left[\frac{1}{4}x^4\ln x - \frac{1}{16}x^4\right]_1^e = \frac{1}{4}e^4 - \frac{1}{16}e^4 - 0 + \frac{1}{16} = \frac{3}{16}e^4 + \frac{1}{16}$

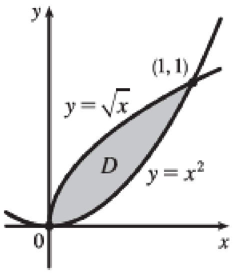
11. $\iint_D y^2e^{xy}\,dA = \int_0^4 \int_0^y y^2e^{xy}\,dx\,dy = \int_0^4 \left[ye^{xy}\right]_{x=0}^{x=y} dy = \int_0^4 \left(ye^{y^2} - y\right) dy$

$= \left[\frac{1}{2}e^{y^2} - \frac{1}{2}y^2\right]_0^4 = \frac{1}{2}e^{16} - 8 - \frac{1}{2} + 0 = \frac{1}{2}e^{16} - \frac{17}{2}$

12. $\int_0^1 \int_0^y x\sqrt{y^2 - x^2}\,dx\,dy = \int_0^1 \left[-\frac{1}{3}(y^2 - x^2)^{3/2}\right]_{x=0}^{x=y} dy = \frac{1}{3}\int_0^1 y^3\,dy = \frac{1}{3}\cdot\left.\frac{1}{4}y^4\right]_0^1 = \frac{1}{12}$

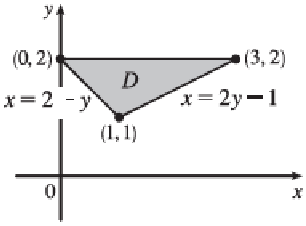
13. $\int_0^1 \int_0^{x^2} x\cos y\,dy\,dx = \int_0^1 \left[x\sin y\right]_{y=0}^{y=x^2} dx = \int_0^1 x\sin x^2\,dx = \left. -\frac{1}{2}\cos x^2\right]_0^1 = \frac{1}{2}(1 - \cos 1)$

14.



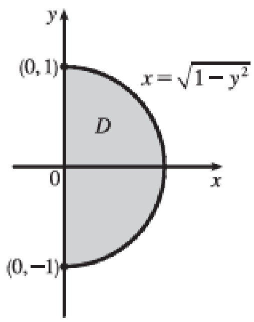
$$\int_0^1 \int_{x^2}^{\sqrt{x}} (x+y)\,dy\,dx = \int_0^1 \left[xy + \tfrac{1}{2}y^2\right]_{y=x^2}^{y=\sqrt{x}} dx$$
$$= \int_0^1 \left(x^{3/2} + \tfrac{1}{2}x - x^3 - \tfrac{1}{2}x^4\right) dx$$
$$= \left[\tfrac{2}{5}x^{5/2} + \tfrac{1}{4}x^2 - \tfrac{1}{4}x^4 - \tfrac{1}{10}x^5\right]_0^1 = \tfrac{3}{10}$$

15.



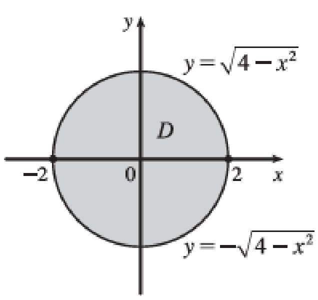
$$\int_1^2 \int_{2-y}^{2y-1} y^3\,dx\,dy = \int_1^2 \left[xy^3\right]_{x=2-y}^{x=2y-1} dy = \int_1^2 \left[(2y-1)-(2-y)\right] y^3\,dy$$
$$= \int_1^2 (3y^4 - 3y^3)\,dy = \left[\tfrac{3}{5}y^5 - \tfrac{3}{4}y^4\right]_1^2$$
$$= \tfrac{96}{5} - 12 - \tfrac{3}{5} + \tfrac{3}{4} = \tfrac{147}{20}$$

16.



$$\iint_D xy^2\,dA = \int_{-1}^1 \int_0^{\sqrt{1-y^2}} xy^2\,dx\,dy$$
$$= \int_{-1}^1 y^2 \left[\tfrac{1}{2}x^2\right]_{x=0}^{x=\sqrt{1-y^2}} dy = \tfrac{1}{2}\int_{-1}^1 y^2(1-y^2)\,dy$$
$$= \tfrac{1}{2}\int_{-1}^1 (y^2 - y^4)\,dy = \tfrac{1}{2}\left[\tfrac{1}{3}y^3 - \tfrac{1}{5}y^5\right]_{-1}^1$$
$$= \tfrac{1}{2}\left(\tfrac{1}{3} - \tfrac{1}{5} + \tfrac{1}{3} - \tfrac{1}{5}\right) = \tfrac{2}{15}$$

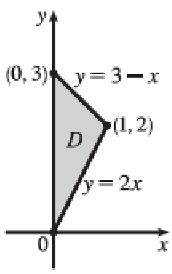
17.



$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (2x - y)\,dy\,dx$$
$$= \int_{-2}^2 \left[2xy - \tfrac{1}{2}y^2\right]_{y=-\sqrt{4-x^2}}^{y=\sqrt{4-x^2}} dx$$
$$= \int_{-2}^2 \left[2x\sqrt{4-x^2} - \tfrac{1}{2}(4-x^2) + 2x\sqrt{4-x^2} + \tfrac{1}{2}(4-x^2)\right] dx$$
$$= \int_{-2}^2 4x\sqrt{4-x^2}\,dx = -\tfrac{4}{3}(4-x^2)^{3/2}\Big]_{-2}^2 = 0$$

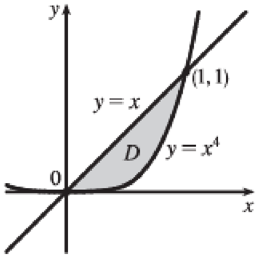
[Or, note that $4x\sqrt{4-x^2}$ is an odd function, so $\int_{-2}^2 4x\sqrt{4-x^2}\,dx = 0$.]

18.



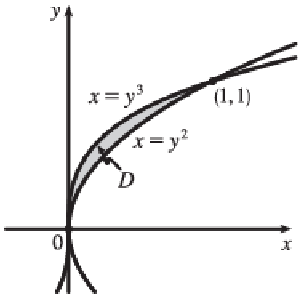
$$\iint_D 2xy\,dA = \int_0^1 \int_{2x}^{3-x} 2xy\,dy\,dx = \int_0^1 \left[xy^2\right]_{y=2x}^{y=3-x} dx$$
$$= \int_0^1 x\left[(3-x)^2 - (2x)^2\right] dx = \int_0^1 (-3x^3 - 6x^2 + 9x)\,dx$$
$$= \left[-\tfrac{3}{4}x^4 - 2x^3 + \tfrac{9}{2}x^2\right]_0^1 = -\tfrac{3}{4} - 2 + \tfrac{9}{2} = \tfrac{7}{4}$$

19.



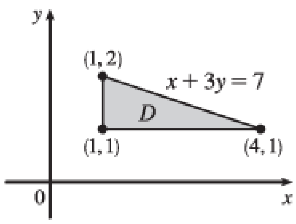
$$V = \int_0^1 \int_{x^4}^{x} (x + 2y)\, dy\, dx$$
$$= \int_0^1 \left[xy + y^2\right]_{y=x^4}^{y=x} dx = \int_0^1 (2x^2 - x^5 - x^8)\, dx$$
$$= \left[\tfrac{2}{3}x^3 - \tfrac{1}{6}x^6 - \tfrac{1}{9}x^9\right]_0^1 = \tfrac{2}{3} - \tfrac{1}{6} - \tfrac{1}{9} = \tfrac{7}{18}$$

20.



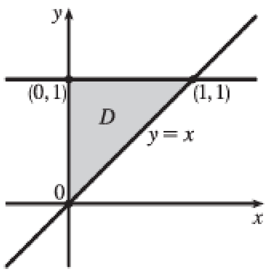
$$V = \int_0^1 \int_{y^3}^{y^2} (2x + y^2)\, dx\, dy$$
$$= \int_0^1 \left[x^2 + xy^2\right]_{x=y^3}^{x=y^2} dy = \int_0^1 (2y^4 - y^6 - y^5)\, dy$$
$$= \left[\tfrac{2}{5}y^5 - \tfrac{1}{7}y^7 - \tfrac{1}{6}y^6\right]_0^1 = \tfrac{19}{210}$$

21.



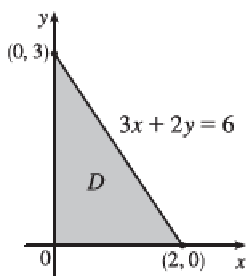
$$V = \int_1^2 \int_1^{7-3y} xy\, dx\, dy = \int_1^2 \left[\tfrac{1}{2}x^2 y\right]_{x=1}^{x=7-3y} dy$$
$$= \tfrac{1}{2}\int_1^2 (48y - 42y^2 + 9y^3)\, dy$$
$$= \tfrac{1}{2}\left[24y^2 - 14y^3 + \tfrac{9}{4}y^4\right]_1^2 = \tfrac{31}{8}$$

22.



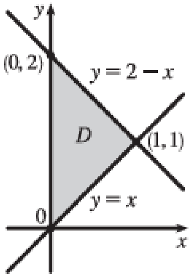
$$V = \int_0^1 \int_x^1 (x^2 + 3y^2)\, dy\, dx$$
$$= \int_0^1 \left[x^2 y + y^3\right]_{y=x}^{y=1} dx = \int_0^1 (x^2 + 1 - 2x^3)\, dx$$
$$= \left[\tfrac{1}{3}x^3 + x - \tfrac{1}{2}x^4\right]_0^1 = \tfrac{5}{6}$$

23.



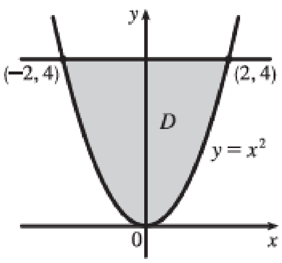
$$V = \int_0^2 \int_0^{3-\frac{3}{2}x} (6 - 3x - 2y)\, dy\, dx$$
$$= \int_0^2 \left[6y - 3xy - y^2\right]_{y=0}^{y=3-\frac{3}{2}x} dx$$
$$= \int_0^2 \left[6(3 - \tfrac{3}{2}x) - 3x(3 - \tfrac{3}{2}x) - (3 - \tfrac{3}{2}x)^2\right] dx$$
$$= \int_0^2 \left(\tfrac{9}{4}x^2 - 9x + 9\right) dx = \left[\tfrac{3}{4}x^3 - \tfrac{9}{2}x^2 + 9x\right]_0^2 = 6 - 0 = 6$$

24.



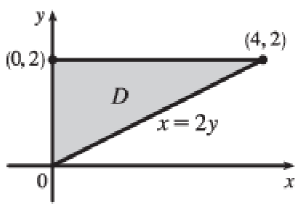
$$V=\int_0^1\int_x^{2-x}x\,dy\,dx$$
$$=\int_0^1 x\left[\,y\,\right]_{y=x}^{y=2-x}\,dx=\int_0^1(2x-2x^2)\,dx$$
$$=\left[x^2-\tfrac{2}{3}x^3\right]_0^1=\tfrac{1}{3}$$

25.



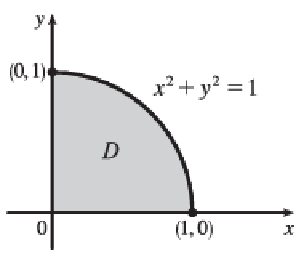
$$V=\int_{-2}^2\int_{x^2}^4 x^2\,dy\,dx$$
$$=\int_{-2}^2 x^2\left[\,y\,\right]_{y=x^2}^{y=4}\,dx=\int_{-2}^2(4x^2-x^4)\,dx$$
$$=\left[\tfrac{4}{3}x^3-\tfrac{1}{5}x^5\right]_{-2}^2=\tfrac{32}{3}-\tfrac{32}{5}+\tfrac{32}{3}-\tfrac{32}{5}=\tfrac{128}{15}$$

26.



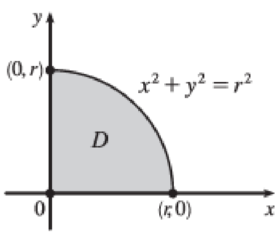
$$V=\int_0^2\int_0^{2y}\sqrt{4-y^2}\,dx\,dy=\int_0^2\left[x\sqrt{4-y^2}\,\right]_{x=0}^{x=2y}dy$$
$$=\int_0^2 2y\sqrt{4-y^2}\,dy=\left[-\tfrac{2}{3}\left(4-y^2\right)^{3/2}\right]_0^2=0+\tfrac{16}{3}=\tfrac{16}{3}$$

27.



$$V=\int_0^1\int_0^{\sqrt{1-x^2}}y\,dy\,dx=\int_0^1\left[\frac{y^2}{2}\right]_{y=0}^{y=\sqrt{1-x^2}}dx$$
$$=\int_0^1\frac{1-x^2}{2}\,dx=\tfrac{1}{2}\left[x-\tfrac{1}{3}x^3\right]_0^1=\tfrac{1}{3}$$

28.

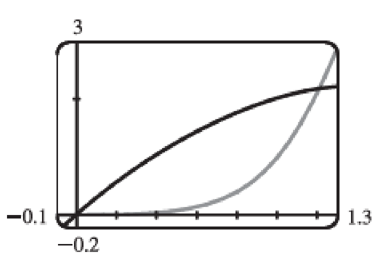


By symmetry, the desired volume V is 8 times the volume V_1 in the first octant. Now

$$V_1=\int_0^r\int_0^{\sqrt{r^2-y^2}}\sqrt{r^2-y^2}\,dx\,dy=\int_0^r\left[x\sqrt{r^2-y^2}\,\right]_{x=0}^{x=\sqrt{r^2-y^2}}dy$$
$$=\int_0^r(r^2-y^2)\,dy=\left[r^2y-\tfrac{1}{3}y^3\right]_0^r=\tfrac{2}{3}r^3$$

Thus $V=\tfrac{16}{3}r^3$.

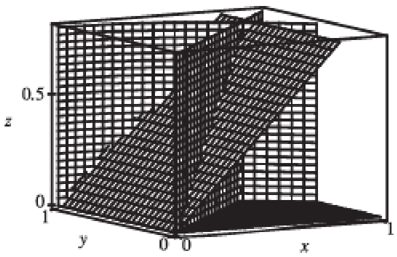
29.

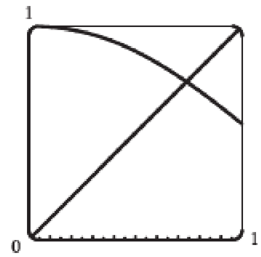


From the graph, it appears that the two curves intersect at $x=0$ and at $x\approx 1.213$. Thus the desired integral is

$$\iint_D x\,dA\approx\int_0^{1.213}\int_{x^4}^{3x-x^2}x\,dy\,dx=\int_0^{1.213}\left[xy\right]_{y=x^4}^{y=3x-x^2}dx$$
$$=\int_0^{1.213}(3x^2-x^3-x^5)\,dx=\left[x^3-\tfrac{1}{4}x^4-\tfrac{1}{6}x^6\right]_0^{1.213}$$
$$\approx 0.713$$

30.





The desired solid is shown in the first graph. From the second graph, we estimate that $y = \cos x$ intersects $y = x$ at $x \approx 0.7391$. Therefore the volume of the solid is

$$V \approx \int_0^{0.7391} \int_x^{\cos x} z\,dy\,dx = \int_0^{0.7391} \int_x^{\cos x} x\,dy\,dx = \int_0^{0.7391} \left[xy\right]_{y=x}^{y=\cos x}\,dx$$

$$= \int_0^{0.7391} (x\cos x - x^2)\,dx = \left[\cos x + x\sin x - \tfrac{1}{3}x^3\right]_0^{0.7391} \approx 0.1024$$

Note: There is a different solid which can also be construed to satisfy the conditions stated in the exercise. This is the solid bounded by all of the given surfaces, as well as the plane $y = 0$. In case you calculated the volume of this solid and want to check your work, its volume is $V \approx \int_0^{0.7391} \int_0^x x\,dy\,dx + \int_{0.7391}^{\pi/2} \int_0^{\cos x} x\,dy\,dx \approx 0.4684$.

31. The two bounding curves $y = 1 - x^2$ and $y = x^2 - 1$ intersect at $(\pm 1, 0)$ with $1 - x^2 \geq x^2 - 1$ on $[-1, 1]$. Within this region, the plane $z = 2x + 2y + 10$ is above the plane $z = 2 - x - y$, so

$$V = \int_{-1}^{1} \int_{x^2-1}^{1-x^2} (2x + 2y + 10)\,dy\,dx - \int_{-1}^{1} \int_{x^2-1}^{1-x^2} (2 - x - y)\,dy\,dx$$

$$= \int_{-1}^{1} \int_{x^2-1}^{1-x^2} (2x + 2y + 10 - (2 - x - y))\,dy\,dx$$

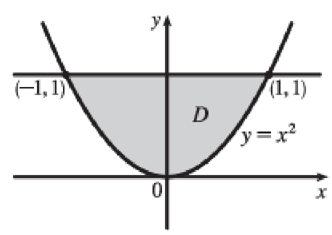
$$= \int_{-1}^{1} \int_{x^2-1}^{1-x^2} (3x + 3y + 8)\,dy\,dx = \int_{-1}^{1} \left[3xy + \tfrac{3}{2}y^2 + 8y\right]_{y=x^2-1}^{y=1-x^2}\,dx$$

$$= \int_{-1}^{1} \left[3x(1 - x^2) + \tfrac{3}{2}(1 - x^2)^2 + 8(1 - x^2) - 3x(x^2 - 1) - \tfrac{3}{2}(x^2 - 1)^2 - 8(x^2 - 1)\right] dx$$

$$= \int_{-1}^{1} (-6x^3 - 16x^2 + 6x + 16)\,dx = \left[-\tfrac{3}{2}x^4 - \tfrac{16}{3}x^3 + 3x^2 + 16x\right]_{-1}^{1}$$

$$= -\tfrac{3}{2} - \tfrac{16}{3} + 3 + 16 + \tfrac{3}{2} - \tfrac{16}{3} - 3 + 16 = \tfrac{64}{3}$$

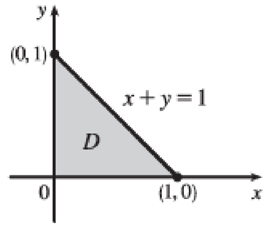
32. The two planes intersect in the line $y = 1$, $z = 3$, so the region of integration is the plane region enclosed by the parabola $y = x^2$ and the line $y = 1$. We have $2 + y \geq 3y$ for $0 \leq y \leq 1$, so the solid region is bounded above by $z = 2 + y$ and bounded below by $z = 3y$.

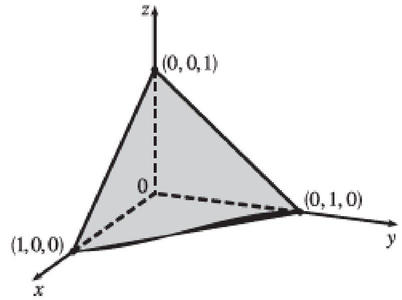


$$V = \int_{-1}^{1} \int_{x^2}^{1} (2 + y)\,dy\,dx - \int_{-1}^{1} \int_{x^2}^{1} (3y)\,dy\,dx = \int_{-1}^{1} \int_{x^2}^{1} (2 + y - 3y)\,dy\,dx = \int_{-1}^{1} \int_{x^2}^{1} (2 - 2y)\,dy\,dx$$

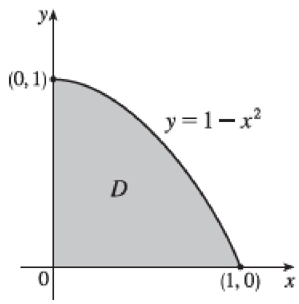
$$= \int_{-1}^{1} \left[2y - y^2\right]_{y=x^2}^{y=1}\,dx = \int_{-1}^{1} (1 - 2x^2 + x^4)\,dx = x - \tfrac{2}{3}x^3 + \tfrac{1}{5}x^5\Big]_{-1}^{1} = \tfrac{16}{15}$$

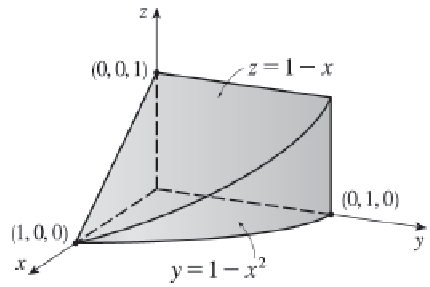
33. The solid lies below the plane $z = 1 - x - y$ or $x + y + z = 1$ and above the region $D = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le 1 - x\}$ in the xy-plane. The solid is a tetrahedron.





34. The solid lies below the plane $z = 1 - x$ and above the region $D = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le 1 - x^2\}$ in the xy-plane.





35. The two bounding curves $y = x^3 - x$ and $y = x^2 + x$ intersect at the origin and at $x = 2$, with $x^2 + x > x^3 - x$ on $(0, 2)$. Using a CAS, we find that the volume is

$$V = \int_0^2 \int_{x^3 - x}^{x^2 + x} z \, dy \, dx = \int_0^2 \int_{x^3 - x}^{x^2 + x} (x^3 y^4 + xy^2) \, dy \, dx = \frac{13{,}984{,}735{,}616}{14{,}549{,}535}$$

36. For $|x| \le 1$ and $|y| \le 1$, $2x^2 + y^2 < 8 - x^2 - 2y^2$. Also, the cylinder is described by the inequalities $-1 \le x \le 1$, $-\sqrt{1 - x^2} \le y \le \sqrt{1 - x^2}$. So the volume is given by

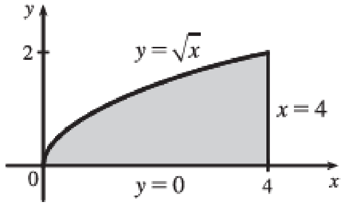
$$V = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} [(8 - x^2 - 2y^2) - (2x^2 + y^2)] \, dy \, dx = \frac{13\pi}{2} \qquad \text{[using a CAS]}$$

37. The two surfaces intersect in the circle $x^2 + y^2 = 1$, $z = 0$ and the region of integration is the disk D: $x^2 + y^2 \le 1$.

Using a CAS, the volume is $\iint_D (1 - x^2 - y^2) \, dA = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (1 - x^2 - y^2) \, dy \, dx = \frac{\pi}{2}$.

38. The projection onto the xy-plane of the intersection of the two surfaces is the circle $x^2 + y^2 = 2y \Rightarrow x^2 + y^2 - 2y = 0 \Rightarrow x^2 + (y - 1)^2 = 1$, so the region of integration is given by $-1 \le x \le 1$, $1 - \sqrt{1 - x^2} \le y \le 1 + \sqrt{1 - x^2}$. In this region, $2y \ge x^2 + y^2$ so, using a CAS, the volume is

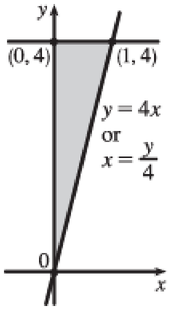
$$V = \int_{-1}^{1} \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} [2y - (x^2 + y^2)] \, dy \, dx = \frac{\pi}{2}$$

39. 

Because the region of integration is

$D = \{(x,y) \mid 0 \le y \le \sqrt{x}, 0 \le x \le 4\} = \{(x,y) \mid y^2 \le x \le 4, 0 \le y \le 2\}$

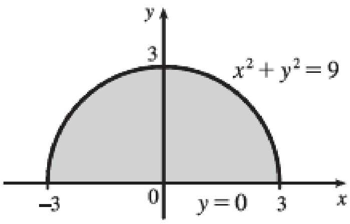
we have $\int_0^4 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx = \iint_D f(x,y)\,dA = \int_0^2 \int_{y^2}^4 f(x,y)\,dx\,dy$.

40. 

Because the region of integration is

$D = \{(x,y) \mid 4x \le y \le 4, 0 \le x \le 1\} = \{(x,y) \mid 0 \le x \le \frac{y}{4}, 0 \le y \le 4\}$

we have $\int_0^1 \int_{4x}^4 f(x,y)\,dy\,dx = \iint_D f(x,y)\,dA = \int_0^4 \int_0^{y/4} f(x,y)\,dx\,dy$.

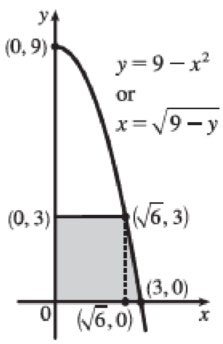
41. 

Because the region of integration is

$$D = \left\{(x,y) \mid -\sqrt{9-y^2} \le x \le \sqrt{9-y^2}, 0 \le y \le 3\right\}$$
$$= \left\{(x,y) \mid 0 \le y \le \sqrt{9-x^2}, -3 \le x \le 3\right\}$$

we have

$$\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x,y)\,dx\,dy = \iint_D f(x,y)\,dA$$
$$= \int_{-3}^3 \int_0^{\sqrt{9-x^2}} f(x,y)\,dy\,dx$$

42. 

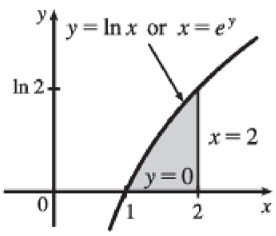
To reverse the order, we must break the region into two separate type I regions.

Because the region of integration is

$$D = \{(x,y) \mid 0 \le x \le \sqrt{9-y}, 0 \le y \le 3\}$$
$$= \{(x,y) \mid 0 \le y \le 3, 0 \le x \le \sqrt{6}\} \cup \{(x,y) \mid 0 \le y \le 9 - x^2, \sqrt{6} \le x \le 3\}$$

we have

$$\int_0^3 \int_0^{\sqrt{9-y}} f(x,y)\,dx\,dy = \iint_D f(x,y)\,dA$$
$$= \int_0^{\sqrt{6}} \int_0^3 f(x,y)\,dy\,dx + \int_{\sqrt{6}}^3 \int_0^{9-x^2} f(x,y)\,dy\,dx$$

43. 

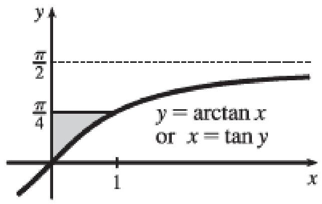
Because the region of integration is

$D = \{(x,y) \mid 0 \le y \le \ln x, 1 \le x \le 2\} = \{(x,y) \mid e^y \le x \le 2, 0 \le y \le \ln 2\}$

we have

$$\int_1^2 \int_0^{\ln x} f(x,y)\,dy\,dx = \iint_D f(x,y)\,dA = \int_0^{\ln 2} \int_{e^y}^2 f(x,y)\,dx\,dy$$

44.



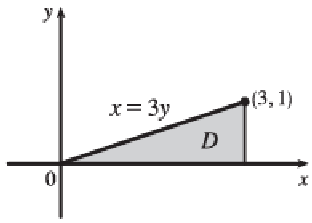
Because the region of integration is

$$D = \left\{(x, y) \mid \arctan x \le y \le \tfrac{\pi}{4}, 0 \le x \le 1\right\}$$
$$= \left\{(x, y) \mid 0 \le x \le \tan y, 0 \le y \le \tfrac{\pi}{4}\right\}$$

we have

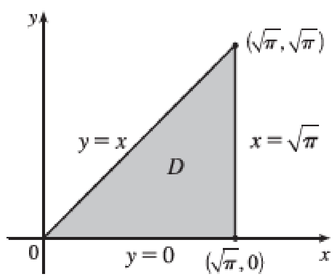
$$\int_0^1 \int_{\arctan x}^{\pi/4} f(x,y)\,dy\,dx = \iint_D f(x,y)\,dA = \int_0^{\pi/4} \int_0^{\tan y} f(x,y)\,dx\,dy$$

45.



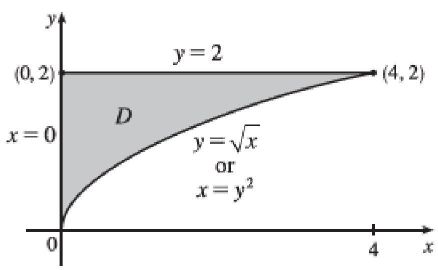
$$\int_0^1 \int_{3y}^3 e^{x^2}\,dx\,dy = \int_0^3 \int_0^{x/3} e^{x^2}\,dy\,dx = \int_0^3 \left[e^{x^2} y\right]_{y=0}^{y=x/3} dx$$
$$= \int_0^3 \left(\frac{x}{3}\right) e^{x^2}\,dx = \tfrac{1}{6}\, e^{x^2}\Big]_0^3 = \frac{e^9 - 1}{6}$$

46.



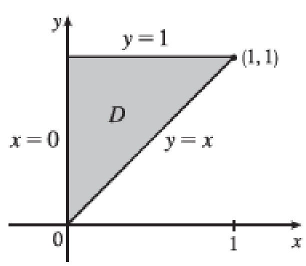
$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)\,dx\,dy = \int_0^{\sqrt{\pi}} \int_0^x \cos(x^2)\,dy\,dx$$
$$= \int_0^{\sqrt{\pi}} \cos(x^2)\,\left[y\right]_{y=0}^{y=x} dx = \int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$$
$$= \tfrac{1}{2}\,\sin(x^2)\Big]_0^{\sqrt{\pi}} = \tfrac{1}{2}(\sin\pi - \sin 0) = 0$$

47.



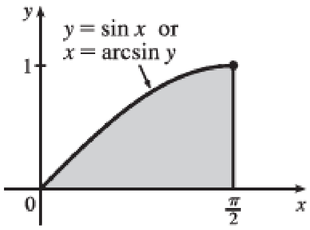
$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\,dy\,dx = \int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\,dx\,dy$$
$$= \int_0^2 \frac{1}{y^3+1}\,\left[x\right]_{x=0}^{x=y^2} dy = \int_0^2 \frac{y^2}{y^3+1}\,dy$$
$$= \tfrac{1}{3}\ln\left|y^3+1\right|\Big]_0^2 = \tfrac{1}{3}(\ln 9 - \ln 1) = \tfrac{1}{3}\ln 9$$

48.



$$\int_0^1 \int_x^1 e^{x/y}\,dy\,dx = \int_0^1 \int_0^y e^{x/y}\,dx\,dy = \int_0^1 \left[ye^{x/y}\right]_{x=0}^{x=y} dy$$
$$= \textstyle\int_0^1 (e-1)y\,dy = \tfrac{1}{2}(e-1)y^2\Big]_0^1$$
$$= \tfrac{1}{2}(e-1)$$

49.



$$\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, dx\, dy$$

$$= \int_0^{\pi/2} \int_0^{\sin x} \cos x \sqrt{1+\cos^2 x}\, dy\, dx$$

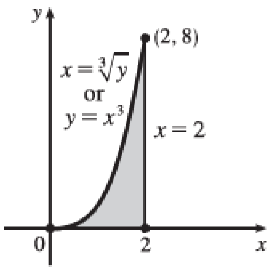
$$= \int_0^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, \Big[\, y \,\Big]_{y=0}^{y=\sin x}\, dx$$

$$= \int_0^{\pi/2} \cos x \sqrt{1+\cos^2 x} \sin x\, dx \qquad \left[\begin{array}{c}\text{Let } u = \cos x,\ du = -\sin x\, dx, \\ dx = du/(-\sin x)\end{array}\right]$$

$$= \int_1^0 -u\sqrt{1+u^2}\, du = -\tfrac{1}{3}\left(1+u^2\right)^{3/2}\Big]_1^0$$

$$= \tfrac{1}{3}\left(\sqrt{8}-1\right) = \tfrac{1}{3}\left(2\sqrt{2}-1\right)$$

50.



$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy = \int_0^2 \int_0^{x^3} e^{x^4}\, dy\, dx$$

$$= \int_0^2 e^{x^4} \Big[\, y \,\Big]_{y=0}^{y=x^3} dx = \int_0^2 x^3 e^{x^4}\, dx$$

$$= \tfrac{1}{4} e^{x^4}\Big]_0^2 = \tfrac{1}{4}\left(e^{16}-1\right)$$