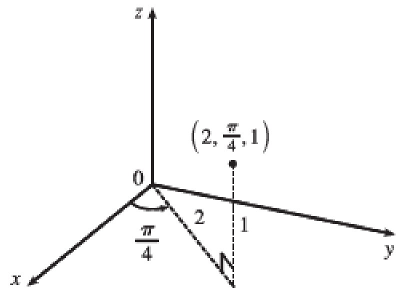
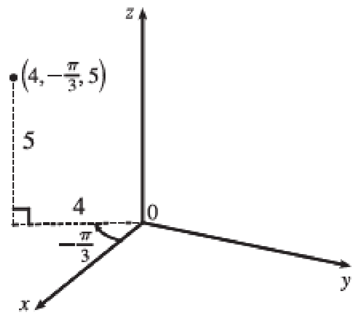
1. (a)



$x = 2\cos\frac{\pi}{4} = \sqrt{2}$, $y = 2\sin\frac{\pi}{4} = \sqrt{2}$, $z = 1$,

so the point is $(\sqrt{2}, \sqrt{2}, 1)$ in rectangular coordinates.

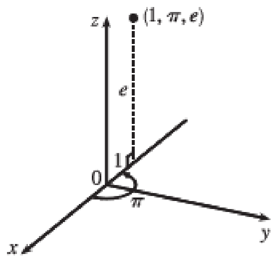
(b)



$x = 4\cos\left(-\frac{\pi}{3}\right) = 2$, $y = 4\sin\left(-\frac{\pi}{3}\right) = -2\sqrt{3}$,

and $z = 5$, so the point is $(2, -2\sqrt{3}, 5)$ in rectangular coordinates.

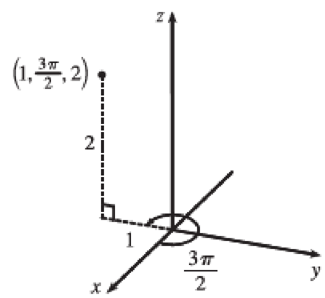
2. (a)



$x = 1\cos\pi = -1$, $y = 1\sin\pi = 0$, and $z = e$,

so the point is $(-1, 0, e)$ in rectangular coordinates.

(b)



$x = 1\cos\frac{3\pi}{2} = 0$, $y = 1\sin\frac{3\pi}{2} = -1$, $z = 2$,

so the point is $(0, -1, 2)$ in rectangular coordinates.

3. (a) $r^2 = x^2 + y^2 = 1^2 + (-1)^2 = 2$ so $r = \sqrt{2}$; $\tan\theta = \frac{y}{x} = \frac{-1}{1} = -1$ and the point $(1, -1)$ is in the fourth quadrant of the xy-plane, so $\theta = \frac{7\pi}{4} + 2n\pi$; $z = 4$. Thus, one set of cylindrical coordinates is $\left(\sqrt{2}, \frac{7\pi}{4}, 4\right)$.

(b) $r^2 = (-1)^2 + \left(-\sqrt{3}\right)^2 = 4$ so $r = 2$; $\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3}$ and the point $\left(-1, -\sqrt{3}\right)$ is in the third quadrant of the xy-plane, so $\theta = \frac{4\pi}{3} + 2n\pi$; $z = 2$. Thus, one set of cylindrical coordinates is $\left(2, \frac{4\pi}{3}, 2\right)$.

4. (a) $r^2 = \left(2\sqrt{3}\right)^2 + 2^2 = 16$ so $r = 4$; $\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$ and the point $\left(2\sqrt{3}, 2\right)$ is in the first quadrant of the xy-plane, so $\theta = \frac{\pi}{6} + 2n\pi$; $z = -1$. Thus, one set of cylindrical coordinates is $\left(4, \frac{\pi}{6}, -1\right)$.

(b) $r^2 = 4^2 + (-3)^2 = 25$ so $r = 5$; $\tan\theta = \frac{-3}{4}$ and the point $(4, -3)$ is in the fourth quadrant of the xy-plane, so $\theta = \tan^{-1}\left(-\frac{3}{4}\right) + 2n\pi \approx -0.64 + 2n\pi$; $z = 2$. Thus, one set of cylindrical coordinates is $\left(5, \tan^{-1}\left(-\frac{3}{4}\right) + 2\pi, 2\right) \approx (5, 5.64, 2)$.

5. Since $\theta = \frac{\pi}{4}$ but r and z may vary, the surface is a vertical half-plane including the z-axis and intersecting the xy-plane in the half-line $y = x$, $x \geq 0$.

6. Since $r = 5$, $x^2 + y^2 = 25$ and the surface is a circular cylinder with radius 5 and axis the z-axis.

7. $z = 4 - r^2 = 4 - (x^2 + y^2)$ or $4 - x^2 - y^2$, so the surface is a circular paraboloid with vertex $(0, 0, 4)$, axis the z-axis, and opening downward.

8. Since $2r^2 + z^2 = 1$ and $r^2 = x^2 + y^2$, we have $2(x^2 + y^2) + z^2 = 1$ or $2x^2 + 2y^2 + z^2 = 1$, an ellipsoid centered at the origin with intercepts $x = \pm\frac{1}{\sqrt{2}}$, $y = \pm\frac{1}{\sqrt{2}}$, $z = \pm 1$.

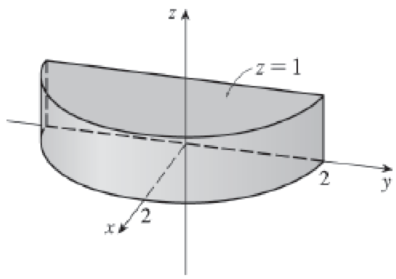
9. (a) $x^2 + y^2 = r^2$, so the equation becomes $z = r^2$.

(b) Substituting $x^2 + y^2 = r^2$ and $y = r\sin\theta$, the equation $x^2 + y^2 = 2y$ becomes $r^2 = 2r\sin\theta$ or $r = 2\sin\theta$.

10. (a) Substituting $x = r\cos\theta$ and $y = r\sin\theta$, the equation $3x + 2y + z = 6$ becomes $3r\cos\theta + 2r\sin\theta + z = 6$ or $z = 6 - r(3\cos\theta + 2\sin\theta)$.

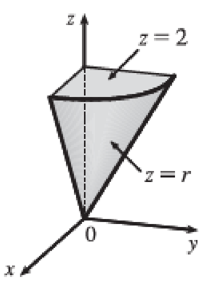
(b) The equation $-x^2 - y^2 + z^2 = 1$ can be written as $-(x^2 + y^2) + z^2 = 1$ which becomes $-r^2 + z^2 = 1$ or $z^2 = 1 + r^2$ in cylindrical coordinates.

11.



$0 \le r \le 2$ and $0 \le z \le 1$ describe a solid circular cylinder with radius 2, axis the z-axis, and height 1, but $-\pi/2 \le \theta \le \pi/2$ restricts the solid to the first and fourth quadrants of the xy-plane, so we have a half-cylinder.

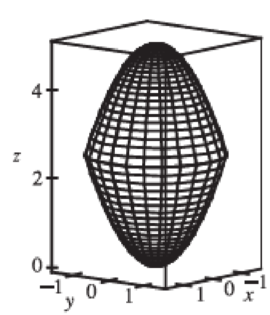
12.

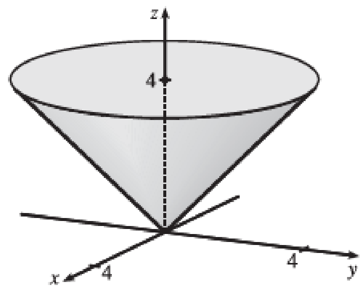


$z = r = \sqrt{x^2 + y^2}$ is a cone that opens upward. Thus $r \le z \le 2$ is the region above this cone and beneath the horizontal plane $z = 2$. $0 \le \theta \le \frac{\pi}{2}$ restricts the solid to that part of this region in the first octant.

13. We can position the cylindrical shell vertically so that its axis coincides with the z-axis and its base lies in the xy-plane. If we use centimeters as the unit of measurement, then cylindrical coordinates conveniently describe the shell as $6 \le r \le 7$, $0 \le \theta \le 2\pi$, $0 \le z \le 20$.

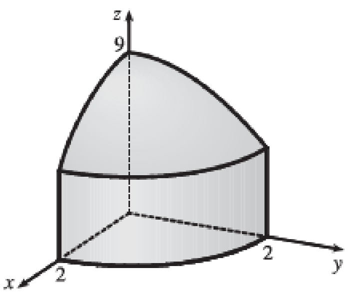
14. In cylindrical coordinates, the equations are $z = r^2$ and $z = 5 - r^2$. The curve of intersection is $r^2 = 5 - r^2$ or $r = \sqrt{5/2}$. So we graph the surfaces in cylindrical coordinates, with $0 \le r \le \sqrt{5/2}$. In Maple, we can use the `coords=cylindrical` option in a regular `plot3d` command. In Mathematica, we can use `ParametricPlot3D`.



15. 

The region of integration is given in cylindrical coordinates by $E = \{(r, \theta, z) \mid 0 \le \theta \le 2\pi, 0 \le r \le 4, r \le z \le 4\}$. This represents the solid region bounded below by the cone $z = r$ and above by the horizontal plane $z = 4$.

$$\int_0^4 \int_0^{2\pi} \int_r^4 r\, dz\, d\theta\, dr = \int_0^4 \int_0^{2\pi} \left[rz\right]_{z=r}^{z=4}\, d\theta\, dr = \int_0^4 \int_0^{2\pi} r(4-r)\, d\theta\, dr$$
$$= \int_0^4 (4r - r^2)\, dr \int_0^{2\pi} d\theta = \left[2r^2 - \tfrac{1}{3}r^3\right]_0^4 \left[\theta\right]_0^{2\pi}$$
$$= \left(32 - \tfrac{64}{3}\right)(2\pi) = \tfrac{64\pi}{3}$$

16. 

The region of integration is given in cylindrical coordinates by $E = \left\{(r, \theta, z) \mid 0 \le \theta \le \pi/2, 0 \le r \le 2, 0 \le z \le 9 - r^2\right\}$. This represents the solid region in the first octant enclosed by the circular cylinder $r = 2$, bounded above by $z = 9 - r^2$, a circular paraboloid, and bounded below by the xy-plane.

$$\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} r\, dz\, dr\, d\theta = \int_0^{\pi/2} \int_0^2 \left[rz\right]_{z=0}^{z=9-r^2}\, dr\, d\theta$$
$$= \int_0^{\pi/2} \int_0^2 r(9 - r^2)\, dr\, d\theta = \int_0^{\pi/2} d\theta \int_0^2 (9r - r^3)\, dr$$
$$= \left[\theta\right]_0^{\pi/2} \left[\tfrac{9}{2}r^2 - \tfrac{1}{4}r^4\right]_0^2 = \tfrac{\pi}{2}(18 - 4) = 7\pi$$

17. In cylindrical coordinates, E is given by $\{(r, \theta, z) \mid 0 \le \theta \le 2\pi, 0 \le r \le 4, -5 \le z \le 4\}$. So

$$\iiint_E \sqrt{x^2 + y^2}\, dV = \int_0^{2\pi} \int_0^4 \int_{-5}^4 \sqrt{r^2}\, r\, dz\, dr\, d\theta = \int_0^{2\pi} d\theta \int_0^4 r^2\, dr \int_{-5}^4 dz$$
$$= \left[\theta\right]_0^{2\pi} \left[\tfrac{1}{3}r^3\right]_0^4 \left[z\right]_{-5}^4 = (2\pi)\left(\tfrac{64}{3}\right)(9) = 384\pi$$

18. The paraboloid $z = 1 - x^2 - y^2$ intersects the xy-plane in the circle $x^2 + y^2 = r^2 = 1$ or $r = 1$, so in cylindrical coordinates, E is given by $\left\{(r, \theta, z) \,\middle|\, 0 \le \theta \le \tfrac{\pi}{2}, 0 \le r \le 1, 0 \le z \le 1 - r^2\right\}$. Thus

$$\iiint_E (x^3 + xy^2)\, dV = \int_0^{\pi/2} \int_0^1 \int_0^{1-r^2} (r^3\cos^3\theta + r^3\cos\theta\sin^2\theta)\, r\, dz\, dr\, d\theta = \int_0^{\pi/2} \int_0^1 \int_0^{1-r^2} r^4\cos\theta\, dz\, dr\, d\theta$$
$$= \int_0^{\pi/2} \int_0^1 r^4\cos\theta \left[z\right]_{z=0}^{z=1-r^2}\, dr\, d\theta = \int_0^{\pi/2} \int_0^1 r^4(1 - r^2)\cos\theta\, dr\, d\theta$$
$$= \int_0^{\pi/2} \cos\theta \left[\tfrac{1}{5}r^5 - \tfrac{1}{7}r^7\right]_{r=0}^{r=1} d\theta = \int_0^{\pi/2} \tfrac{2}{35}\cos\theta\, d\theta = \tfrac{2}{35}\left[\sin\theta\right]_0^{\pi/2} = \tfrac{2}{35}$$

19. In cylindrical coordinates E is bounded by the paraboloid $z = 1 + r^2$, the cylinder $r^2 = 5$ or $r = \sqrt{5}$, and the xy-plane, so E is given by $\left\{(r, \theta, z) \mid 0 \le \theta \le 2\pi, 0 \le r \le \sqrt{5}, 0 \le z \le 1 + r^2\right\}$. Thus

$$\iiint_E e^z\, dV = \int_0^{2\pi} \int_0^{\sqrt{5}} \int_0^{1+r^2} e^z\, r\, dz\, dr\, d\theta = \int_0^{2\pi} \int_0^{\sqrt{5}} r\left[e^z\right]_{z=0}^{z=1+r^2}\, dr\, d\theta = \int_0^{2\pi} \int_0^{\sqrt{5}} r(e^{1+r^2} - 1)\, dr\, d\theta$$
$$= \int_0^{2\pi} d\theta \int_0^{\sqrt{5}} \left(re^{1+r^2} - r\right) dr = 2\pi\left[\tfrac{1}{2}e^{1+r^2} - \tfrac{1}{2}r^2\right]_0^{\sqrt{5}} = \pi(e^6 - e - 5)$$

20. In cylindrical coordinates E is bounded by the planes $z = 0$, $z = r\cos\theta + r\sin\theta + 5$ and the cylinders $r = 2$ and $r = 3$, so E is given by $\{(r,\theta,z) \mid 0 \le \theta \le 2\pi, 2 \le r \le 3, 0 \le z \le r\cos\theta + r\sin\theta + 5\}$. Thus

$$\begin{aligned}\iiint_E x\,dV &= \int_0^{2\pi}\int_2^3\int_0^{r\cos\theta + r\sin\theta+5}(r\cos\theta)\,r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_2^3 (r^2\cos\theta)\big[\,z\,\big]_{z=0}^{z=r\cos\theta+r\sin\theta+5}\,dr\,d\theta \\ &= \int_0^{2\pi}\int_2^3 (r^2\cos\theta)(r\cos\theta + r\sin\theta + 5)\,dr\,d\theta = \int_0^{2\pi}\int_2^3 (r^3(\cos^2\theta + \cos\theta\sin\theta) + 5r^2\cos\theta)\,dr\,d\theta \\ &= \int_0^{2\pi}\left[\tfrac14 r^4(\cos^2\theta + \cos\theta\sin\theta) + \tfrac53 r^3\cos\theta\right]_{r=2}^{r=3} d\theta \\ &= \int_0^{2\pi}\left[\left(\tfrac{81}{4} - \tfrac{16}{4}\right)(\cos^2\theta + \cos\theta\sin\theta) + \tfrac53(27-8)\cos\theta\right] d\theta \\ &= \int_0^{2\pi}\left(\tfrac{65}{4}\left(\tfrac12(1+\cos 2\theta) + \cos\theta\sin\theta\right) + \tfrac{95}{3}\cos\theta\right) d\theta = \left[\tfrac{65}{8}\theta + \tfrac{65}{16}\sin 2\theta + \tfrac{65}{8}\sin^2\theta + \tfrac{95}{3}\sin\theta\right]_0^{2\pi} = \tfrac{65}{4}\pi\end{aligned}$$

21. In cylindrical coordinates, E is bounded by the cylinder $r = 1$, the plane $z = 0$, and the cone $z = 2r$. So $E = \{(r,\theta,z) \mid 0 \le \theta \le 2\pi, 0 \le r \le 1, 0 \le z \le 2r\}$ and

$$\begin{aligned}\iiint_E x^2\,dV &= \int_0^{2\pi}\int_0^1\int_0^{2r} r^2\cos^2\theta\,r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 \left[r^3\cos^2\theta\,z\right]_{z=0}^{z=2r}\,dr\,d\theta = \int_0^{2\pi}\int_0^1 2r^4\cos^2\theta\,dr\,d\theta \\ &= \int_0^{2\pi}\left[\tfrac25 r^5\cos^2\theta\right]_{r=0}^{r=1} d\theta = \tfrac25\int_0^{2\pi}\cos^2\theta\,d\theta = \frac25\int_0^{2\pi}\frac{1+\cos 2\theta}{2}\,d\theta = \frac15\left[\theta + \frac12\sin 2\theta\right]_0^{2\pi} = \frac{2\pi}{5}\end{aligned}$$

22. In cylindrical coordinates E is the solid region within the cylinder $r = 1$ bounded above and below by the sphere $r^2 + z^2 = 4$, so $E = \left\{(r,\theta,z) \mid 0 \le \theta \le 2\pi, 0 \le r \le 1, -\sqrt{4-r^2} \le z \le \sqrt{4-r^2}\right\}$. Thus the volume is

$$\begin{aligned}\iiint_E dV &= \int_0^{2\pi}\int_0^1\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 2r\sqrt{4-r^2}\,dr\,d\theta \\ &= \int_0^{2\pi} d\theta\int_0^1 2r\sqrt{4-r^2}\,dr = 2\pi\left[-\tfrac23(4-r^2)^{3/2}\right]_0^1 = \tfrac43\pi(8 - 3^{3/2})\end{aligned}$$

23. (a) The paraboloids intersect when $x^2 + y^2 = 36 - 3x^2 - 3y^2 \;\Rightarrow\; x^2 + y^2 = 9$, so the region of integration is $D = \left\{(x,y) \mid x^2 + y^2 \le 9\right\}$. Then, in cylindrical coordinates,

$E = \left\{(r,\theta,z) \mid r^2 \le z \le 36 - 3r^2, 0 \le r \le 3, 0 \le \theta \le 2\pi\right\}$ and

$$V = \int_0^{2\pi}\int_0^3\int_{r^2}^{36-3r^2} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^3 \left(36r - 4r^3\right)dr\,d\theta = \int_0^{2\pi}\left[18r^2 - r^4\right]_{r=0}^{r=3} d\theta = \int_0^{2\pi} 81\,d\theta = 162\pi.$$

(b) For constant density K, $m = KV = 162\pi K$ from part (a). Since the region is homogeneous and symmetric, $M_{yz} = M_{xz} = 0$ and

$$\begin{aligned}M_{xy} &= \int_0^{2\pi}\int_0^3\int_{r^2}^{36-3r^2}(zK)\,r\,dz\,dr\,d\theta = K\int_0^{2\pi}\int_0^3 r\left[\tfrac12 z^2\right]_{z=r^2}^{z=36-3r^2} dr\,d\theta \\ &= \tfrac{K}{2}\int_0^{2\pi}\int_0^3 r((36-3r^2)^2 - r^4)\,dr\,d\theta = \tfrac{K}{2}\int_0^{2\pi} d\theta\int_0^3 (8r^5 - 216r^3 + 1296r)\,dr \\ &= \tfrac{K}{2}(2\pi)\left[\tfrac86 r^6 - \tfrac{216}{4}r^4 + \tfrac{1296}{2}r^2\right]_0^3 = \pi K(2430) = 2430\pi K\end{aligned}$$

Thus $(\overline{x},\overline{y},\overline{z}) = \left(\dfrac{M_{yz}}{m}, \dfrac{M_{xz}}{m}, \dfrac{M_{xy}}{m}\right) = \left(0, 0, \frac{2430\pi K}{162\pi K}\right) = (0,0,15)$.

24. (a) $V = \int_{-\pi/2}^{\pi/2} \int_0^{a\cos\theta} \int_{-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} r\, dz\, dr\, d\theta$

$= 4\int_0^{\pi/2} \int_0^{a\cos\theta} \int_0^{\sqrt{a^2-r^2}} r\, dz\, dr\, d\theta$

$= 4\int_0^{\pi/2} \int_0^{a\cos\theta} r\sqrt{a^2-r^2}\, dr\, d\theta$

$= -\frac{4}{3}\int_0^{\pi/2} \left[\left(a^2-r^2\right)^{3/2}\right]_{r=0}^{r=a\cos\theta} d\theta$

$= -\frac{4}{3}\int_0^{\pi/2} \left[\left(a^2-a^2\cos^2\theta\right)^{3/2} - a^3\right] d\theta$

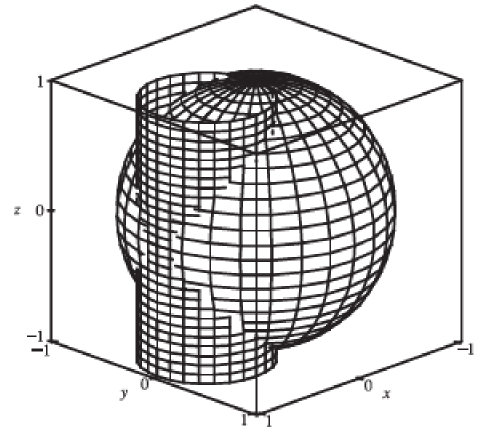
$= -\frac{4}{3}\int_0^{\pi/2} \left[\left(a^2\sin^2\theta\right)^{3/2} - a^3\right] d\theta$

$= -\frac{4}{3}\int_0^{\pi/2} \left(a^3\sin^3\theta - a^3\right) d\theta$

$= -\dfrac{4a^3}{3}\displaystyle\int_0^{\pi/2} \left[\sin\theta\left(1-\cos^2\theta\right) - 1\right] d\theta$

$= -\dfrac{4a^3}{3}\left[-\cos\theta + \frac{1}{3}\cos^3\theta - \theta\right]_0^{\pi/2} = -\dfrac{4a^3}{3}\left(-\frac{\pi}{2}+\frac{2}{3}\right) = \frac{2}{9}a^3(3\pi-4)$

(b)



To plot the cylinder and the sphere on the same screen in Maple, we can use the sequence of commands

```
sphere:=plot3d(sqrt(1-z^2),theta=0..2*Pi,z=-1..1,coords=cylindrical):
cylinder:=plot3d(cos(theta),theta=-Pi/2..Pi/2,z=-1..1,coords=cylindrical):
with(plots): display3d({sphere,cylinder});
```

In Mathematica, we can use

```
sphere=ParametricPlot3D[{Sqrt[1-z^2]*Cos[theta],Sqrt[1-z^2]*Sin[theta],z},
                              {theta,0,2Pi},{z-1,1}]
cylinder=ParametricPlot3D[{(Cos[theta])^2,Cos[theta]*Sin[theta],z},
                              {theta,-Pi/2,Pi/2},{z,-1,1}]
Show[{sphere,cylinder}]
```

25. The paraboloid $z = 4x^2 + 4y^2$ intersects the plane $z = a$ when $a = 4x^2 + 4y^2$ or $x^2 + y^2 = \frac{1}{4}a$. So, in cylindrical coordinates, $E = \left\{(r,\theta,z) \mid 0 \le r \le \frac{1}{2}\sqrt{a}, 0 \le \theta \le 2\pi, 4r^2 \le z \le a\right\}$. Thus

$$m = \int_0^{2\pi} \int_0^{\sqrt{a}/2} \int_{4r^2}^{a} Kr\, dz\, dr\, d\theta = K\int_0^{2\pi} \int_0^{\sqrt{a}/2} \left(ar - 4r^3\right) dr\, d\theta$$

$$= K\int_0^{2\pi} \left[\tfrac{1}{2}ar^2 - r^4\right]_{r=0}^{r=\sqrt{a}/2} d\theta = K\int_0^{2\pi} \tfrac{1}{16}a^2\, d\theta = \tfrac{1}{8}a^2\pi K$$

Since the region is homogeneous and symmetric, $M_{yz} = M_{xz} = 0$ and

$$M_{xy} = \int_0^{2\pi} \int_0^{\sqrt{a}/2} \int_{4r^2}^{a} Krz\, dz\, dr\, d\theta = K\int_0^{2\pi} \int_0^{\sqrt{a}/2} \left(\tfrac{1}{2}a^2r - 8r^5\right) dr\, d\theta$$

$$= K\int_0^{2\pi} \left[\tfrac{1}{4}a^2r^2 - \tfrac{4}{3}r^6\right]_{r=0}^{r=\sqrt{a}/2} d\theta = K\int_0^{2\pi} \tfrac{1}{24}a^3\, d\theta = \tfrac{1}{12}a^3\pi K$$

Hence $(\overline{x},\overline{y},\overline{z}) = \left(0,0,\frac{2}{3}a\right)$.

26. Since density is proportional to the distance from the z-axis, we can say $\rho(x,y,z) = K\sqrt{x^2+y^2}$. Then

$$m = 2\int_0^{2\pi}\int_0^a\int_0^{\sqrt{a^2-r^2}} Kr^2\,dz\,dr\,d\theta = 2K\int_0^{2\pi}\int_0^a r^2\sqrt{a^2-r^2}\,dr\,d\theta$$
$$= 2K\int_0^{2\pi}\left[\tfrac{1}{8}r(2r^2-a^2)\sqrt{a^2-r^2}+\tfrac{1}{8}a^4\sin^{-1}(r/a)\right]_{r=0}^{r=a}d\theta = 2K\int_0^{2\pi}\left[\left(\tfrac{1}{8}a^4\right)\left(\tfrac{\pi}{2}\right)\right]d\theta = \tfrac{1}{4}a^4\pi^2K$$

27. The region of integration is the region above the cone $z=\sqrt{x^2+y^2}$, or $z=r$, and below the plane $z=2$. Also, we have $-2\le y\le 2$ with $-\sqrt{4-y^2}\le x\le\sqrt{4-y^2}$ which describes a circle of radius 2 in the xy-plane centered at $(0,0)$. Thus,

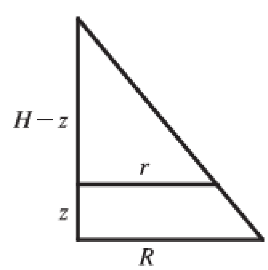
$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\,dz\,dx\,dy = \int_0^{2\pi}\int_0^2\int_r^2 (r\cos\theta)\,z\,r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^2\int_r^2 r^2(\cos\theta)\,z\,dz\,dr\,d\theta$$
$$= \int_0^{2\pi}\int_0^2 r^2(\cos\theta)\left[\tfrac{1}{2}z^2\right]_{z=r}^{z=2}dr\,d\theta = \tfrac{1}{2}\int_0^{2\pi}\int_0^2 r^2(\cos\theta)\left(4-r^2\right)dr\,d\theta$$
$$= \tfrac{1}{2}\int_0^{2\pi}\cos\theta\,d\theta\int_0^2\left(4r^2-r^4\right)dr = \tfrac{1}{2}\left[\sin\theta\right]_0^{2\pi}\left[\tfrac{4}{3}r^3-\tfrac{1}{5}r^5\right]_0^2 = 0$$

28. The region of integration is the region above the plane $z=0$ and below the paraboloid $z=9-x^2-y^2$. Also, we have $-3\le x\le 3$ with $0\le y\le\sqrt{9-x^2}$ which describes the upper half of a circle of radius 3 in the xy-plane centered at $(0,0)$. Thus,

$$\int_{-3}^{3}\int_0^{\sqrt{9-x^2}}\int_0^{9-x^2-y^2}\sqrt{x^2+y^2}\,dz\,dy\,dx = \int_0^{\pi}\int_0^3\int_0^{9-r^2}\sqrt{r^2}\,r\,dz\,dr\,d\theta = \int_0^{\pi}\int_0^3\int_0^{9-r^2} r^2\,dz\,dr\,d\theta$$
$$= \int_0^{\pi}\int_0^3 r^2\left(9-r^2\right)dr\,d\theta = \int_0^{\pi}d\theta\int_0^3\left(9r^2-r^4\right)dr$$
$$= \left[\theta\right]_0^{\pi}\left[3r^3-\tfrac{1}{5}r^5\right]_0^3 = \pi\left(81-\tfrac{243}{5}\right) = \tfrac{162}{5}\pi$$

29. (a) The mountain comprises a solid conical region C. The work done in lifting a small volume of material ΔV with density $g(P)$ to a height $h(P)$ above sea level is $h(P)g(P)\,\Delta V$. Summing over the whole mountain we get $W=\iiint_C h(P)g(P)\,dV$.

(b) Here C is a solid right circular cone with radius $R = 62{,}000$ ft, height $H = 12{,}400$ ft, and density $g(P) = 200\text{ lb/ft}^3$ at all points P in C. We use cylindrical coordinates:



$$\frac{r}{R} = \frac{H-z}{H} = 1-\frac{z}{H}$$

$$W = \int_0^{2\pi}\int_0^H\int_0^{R(1-z/H)} z\cdot 200r\,dr\,dz\,d\theta = 2\pi\int_0^H 200z\left[\tfrac{1}{2}r^2\right]_{r=0}^{r=R(1-z/H)}dz$$
$$= 400\pi\int_0^H z\frac{R^2}{2}\left(1-\frac{z}{H}\right)^2dz = 200\pi R^2\int_0^H\left(z-\frac{2z^2}{H}+\frac{z^3}{H^2}\right)dz$$
$$= 200\pi R^2\left[\frac{z^2}{2}-\frac{2z^3}{3H}+\frac{z^4}{4H^2}\right]_0^H = 200\pi R^2\left(\frac{H^2}{2}-\frac{2H^2}{3}+\frac{H^2}{4}\right)$$
$$= \tfrac{50}{3}\pi R^2H^2 = \tfrac{50}{3}\pi(62{,}000)^2(12{,}400)^2 \approx 3.1\times 10^{19}\text{ ft-lb}$$