進階代數(上) 第十一次作業

上課老師: 翁志文

2008 年十二月十一日

1. (葉彬) Determine all the $n \times n$ matrices in rational canonical form over $\mathbb{R}$ with minimal polynomial $\lambda^2 + 1$.

2. (林家銘) Prove the injective case in The Short Five Lemma.

3. (The Five Lemma) Suppose we have the following commutative diagram of R-module homomorphisms with exact rows.

$$\begin{array}{ccccccccc} M_1 & \longrightarrow & M_2 & \longrightarrow & M_3 & \longrightarrow & M_4 & \longrightarrow & M_5 \\ f_1 \downarrow & \circlearrowright & f_2 \downarrow & \circlearrowright & f_3 \downarrow & \circlearrowright & f_4 \downarrow & \circlearrowright & f_5 \downarrow \\ N_1 & \longrightarrow & N_2 & \longrightarrow & N_3 & \longrightarrow & N_4 & \longrightarrow & N_5 \end{array}$$

   (a) (林育生) Prove directly f_1 surjection and f_2, f_4 injection $\Longrightarrow$ f_3 injection.

   (b) (陳健文) Prove (a) by using The Short Five Lemma.

   (c) (羅健鋒) Prove directly f_5 injection and f_2, f_4 surjection $\Longrightarrow$ f_3 surjection.

   (d) (何昕暘) Prove (c) by using The Short Five Lemma.

4. (賴德展) Suppose

$$0 \longrightarrow M_1 \xrightarrow{f} M \xrightarrow{g} M_2 \longrightarrow 0$$

   and there exists $k : M \to M_1$ such that $kf = 1_{M_1}$. Show M is isomorphic to $M_1 \oplus M_2$.

5. Let P_i, $i \in I$, be R-modules.

   (a) (洪湧昇) Show that if $\oplus_{i \in I} P_i$ is projective then P_i is projective for each $i \in I$.

   (b) (林志峰) Show that if P_i is projective for each $i \in I$ then $\oplus_{i \in I} P_i$ is projective.

6. (呂融昇) Show that $\mathbb{Q}$ is not a projective $\mathbb{Z}$-module.