進階代數(上) 第七次作業

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1. (何昕暘) Let A be an $n \times n$ matrix over $\mathbb{R}$. For $f(\lambda) \in \mathbb{R}[\lambda]$ and $u \in \mathbb{R}^n$, define $f(\lambda)u = f(A)u$ and hence R^n is a $\mathbb{R}[\lambda]$-module. A subspace W of $\mathbb{R}^n$ is A-*invariant* if $AW \subseteq W$. Show that a subspace W of R^n is a $R[\lambda]$-submodule if and only if W is A-invariant.

2. (賴德展) Show that the $\mathbb{Z}$-module $\dfrac{\mathbb{Z} \times \mathbb{Z}}{((2,2),(4,-2))}$ is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_6$. (Hint. $((1,1)) + ((1,0)) = \mathbb{Z} \times \mathbb{Z}$)

3. (洪湧昇) Let $M_1, M_2, \ldots, M_n$ be R-submodule of M such that $M = M_1 + M_2 + \cdots + M_n$ and
$$M_i \cap (M_1 + \cdots + M_{i-1} + M_{i+1} + \cdots + M_n) = 0$$
for each $1 \leq i \leq n$. Show that M is isomorphic to $M_1 \oplus \cdots \oplus M_n$.

4. (林志峰) Let N_1, N_2 be submodules of M_1, M_2 respectively. Show that $N_1 \oplus N_2$ is a submodule of $M_1 \oplus M_2$, and
$$\frac{M_1 \oplus M_2}{N_1 \oplus N_2} \cong \frac{M_1}{N_1} \oplus \frac{M_2}{N_2}.$$

5. (呂融昇) Let $f : M \to N$ and $g : N \to M$ be R-module homomorphisms such that fg is the identity map on N. Show that M is isomorphic to $\mathrm{Ker}\, f \oplus \mathrm{Img}\, g$.

6. (羅元勳) Obtain a Smith normal form for the integral matrix
$$A = \begin{pmatrix} 6 & 2 & 3 & 0 \\ 2 & 3 & -4 & 1 \\ -3 & 3 & 1 & 2 \\ -1 & 2 & -3 & 5 \end{pmatrix}$$
and the corresponding invertible matrices P, Q such that PAQ is in Smith normal form.

7. (連敏筠) Obtain a a Smith normal form for the rational polynomial matrix
$$A = \begin{pmatrix} \lambda - 17 & 8 & 12 & -14 \\ -46 & \lambda + 22 & 35 & -41 \\ 2 & -1 & \lambda - 4 & 4 \\ -4 & 2 & 2 & \lambda - 3 \end{pmatrix}$$
and the corresponding invertible matrices P, Q such that PAQ is in Smith normal form.