1.3 **Cyclic groups**

Definition 1 *G is cyclic if $G = \langle a \rangle$ for some $a \in G$.*

ex.$Z = \langle 1 \rangle = \langle -1 \rangle$ under $+$.

Theorem 1 *Let G be a cyclic group. Then G is isomorphic to Z or Z_n for some $n \in N$, under addition.*
Proof. Suppose $G = \langle a \rangle = \{a^i | i \in Z\}$, for some $a \in G$.
Define $f : Z \to G$ by $f(i) = a^i$
f is clear to be an epimorphism.
If f is isomorphism, then G is isomorphic to Z.
Suppose f is not isomorphism:
Let n be the least integer $n = j - i$ such that $j > i$ and $a^j = a^i$
Note $G = \{a^0, a^1, \cdots, a^{n-1}\}$ and $|G| = n$.
Hence G is isomorphic to Z_n.