Theorem 1. Let G_1, G_2 be *abelian* groups. Then there exist a unique *abelian* group G,up to isomorphism,with homomorphism $\psi_i : G_i \to G$ satisfying the following *coproduct rule* : For any abelian group G' with any homomorphisms $\psi_i' : G_i \to G'$ there exists a unique homomorphism $\psi : G \to G'$ such that $\psi\psi_i = \psi_i'$ for i=1,2. In fact , G is isomorphism to $G_1 \times G_2$.

Proof:
"existence":
Set $G = G_1 \times G_2$ and $\psi_i = \imath_i : G_i \to G_1 \times G_2$ be the inclusive map.
Pick any (G', ψ_1', ψ_2') as described.
Set $\psi : G_1 \times G_2 \to G'$ by $\psi(g_1, g_2) = \psi_1'(g_1) \cdot \psi_2'(g_2)$ for $g_1 \epsilon G_1, g_2 \epsilon G_2$.
Clearly, $\psi\psi_i = \psi_i'$
We need to check that $\psi : G_1 \times G_2 \to G'$ is a homomorphism.

$$\begin{aligned}
\psi((g_1, g_2) \cdot (h_1, h_2)) &= \psi(g_1 h_1, g_2 h_2) \\
&= \psi_1'(g_1 h_1)\psi_2'(g_2 h_2) \\
&= \psi_1'(g_1)\psi_1'(h_1)\psi_2'(g_2)\psi_2'(h_2) \\
&= \psi_1'(g_1)\psi_2'(g_2)\psi_1'(h_1)\psi_2'(h_2) \\
&= \psi(g_1 g_2)\psi(h_1 h_2) \ for\ any\ g_1, h_1 \epsilon G_1
\end{aligned}$$

"uniqueness":
As before.

Question:What is the group G in previous theorem. If we drop all the abelian assumption.