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1.9 Free group

Def：Let X be a set of symbols and

$$F(x) := \left\{ a_1^{n_1} \cdot a_2^{n_2} \cdot a_3^{n_3} \ldots . a_t^{n_t} \middle| a_i \in X, n_i \in Z - \{0\}, a_i \neq a_{i+1} \right\}$$

is the set of "reduced words" of $X \cup X^{-1}$.

We define the product of 2 reduced words by concatenation and reduceing the new word to be in $F(x)$.

Example： $X = \{a,b\}$

$$\left(a^2ba^{-3}b^3a^2\right)\left(a^{-2}b^{-3}a^3b^2a\right)$$
$$= a^2ba^{-3}b^3a^2a^{-2}b^{-3}a^3b^2a$$
$$= a^2b^3a$$

Theorem：① $F(x)$ is a group with $e = \phi$

② Let $i : X \to G$ be an injective map.

Then there exists a unique homomorphism $\varphi : F(x) \to G$ such that $\varphi(a) = i(a)$ if $a \in X$.

pf :Routine.

Note： $F(x)$ is called the free group with generator set X.

Example： $X = \{a\}$ $F(x) = \left\{a^i \middle| i \in Z\right\} \cong \langle Z, + \rangle$

Def：For $Y \subseteq F(x)$

Let N_Y be the smallest normal subgroup containing.

(i.e. $N_Y = \bigcap_{Y \subseteq N \lhd G} N = \left\langle g^{-1}Yg \middle| g \in F(x) \right\rangle$)

$\dfrac{F(x)}{N_Y}$ is called the group generated by X subject to $y = e$ for $y \in Y$.

Example： Fix $n \in N$

Find the group generated by $X = \{a\}$ subject to $a^n = e$.

Sol： $Y = \{a^n\}$ $N_Y = \{a^{in} \mid i \in Z\}$ $F(x) = \{a^i \mid i \in Z\}$

$$\frac{F(x)}{N_Y} = \{\overline{a^0}, \overline{a^1}, \overline{a^2}, \ldots\ldots \overline{a^{n-1}}\} \cong \langle Z_n, + \rangle$$