ex:

f(x)= x^4 - 2 $\in \mathbb{Q}[X]$

f(x)= 0 $\Rightarrow$ x = $\sqrt[4]{2}$, $\sqrt[4]{2}i$, -$\sqrt[4]{2}$, -i$\sqrt[4]{2}$

Q(f) = Q($\sqrt[4]{2}$, i)

$\sigma \in$ G(Q(f)/Q) $\Rightarrow$ $\sigma(\sqrt[4]{2})$ = $\sqrt[4]{2}$, $\sqrt[4]{2}$i, -$\sqrt[4]{2}$, -i$\sqrt[4]{2}$

σ(i) = i, -i (x^2+1 = 0)

Say $\sigma \in$ Q(Q(f)/Q) and

$$\begin{cases} \tau(\sqrt[4]{2}) = \sqrt[4]{2} \\ \tau(i) = -i \end{cases}$$

Then | τ | = 2

Note $\sigma\tau(\sqrt[4]{2})$ = $\sqrt[4]{2}$i = $\tau\sigma^3(\sqrt[4]{2})$

Hence G(Q(f)/Q) = $< \sigma, \tau > = D_4$

Q($\sqrt[4]{2}$, i) is a field extension of Q(i, $\sqrt{2}$), Q($\sqrt[4]{2}$-$\sqrt[4]{2}$i), Q($\sqrt[4]{2}$+$\sqrt[4]{2}$i), Q($\sqrt[4]{2}$), Q($\sqrt[4]{2}$i).

Q(i, $\sqrt{2}$) is a field extension of Q(i), Q($\sqrt{2}$i), Q($\sqrt{2}$).

Q($\sqrt[4]{2}$) is a field extension of Q($\sqrt{2}$).

Q($\sqrt[4]{2}$i) is a field extension of Q($\sqrt{2}$).

Q(i) and Q($\sqrt{2}$i) are field extension of Q.

$< e >$ is a field extension of $< \sigma^2 >$, $< \tau\sigma >$, $< \tau\sigma_3 >$, $< \tau >$, $< \tau\sigma^2 >$.

$< \sigma^2 >$ is a field extension of $< \sigma >$ and $\{e, \sigma^2, \tau\sigma, \tau\sigma^3\}$.

$< \tau\sigma >$ is a field extension of $\{e, \sigma^2, \tau\sigma, \tau\sigma^3\}$.

$< \tau\sigma_3 >$ is a field extension of $\{e, \sigma^2, \tau\sigma, \tau\sigma^3\}$.

$< \tau >$ is a field extension of $\{e, \sigma^2, \tau, \tau\sigma^3\}$

$< \tau\sigma^2 >$ is a field extension of $\{e, \sigma^2, \tau, \tau\sigma^3\}$

$< \tau >$, $\{e, \sigma^2, \tau\sigma, \tau\sigma^3\}$ and $\{e, \sigma^2, \tau, \tau\sigma^3\}$ are field extension of G(Q(f)/Q) = $< \sigma, \tau >$ = D_4