六月十八日上課筆記

Let y_i be indeterminats for $1 \leq i \leq 5$.
Set

$$\begin{aligned} s_1 &= \sum_{i=1}^{5} y_i \\ s_1 &= \sum_{1 \leq i < j \leq 5} y_i y_j \\ &\vdots \\ s_5 &= y_1 y_2 y_3 y_4 y_5 \end{aligned}$$

and

$$\begin{aligned} f(x) &= x^5 - s_1 x^4 + s_2 x^3 - s_3 x^2 + s_4 x - s_5 \\ &= (x - y_1)(x - y_2)(x - y_3)(x - y_4)(x - y_5) \end{aligned}$$

Theorem. K(f) is not a radical extension over $K = \mathbb{Q}(s_1, s_2, \cdots, s_5)$.

Proof. Note $K(f) = K(y_1, y_2, y_3, y_4, y_5)$
Each $\sigma \in G\left(K(f)\Big/_K\right)$, σ permutes roots y_1, y_2, y_3, y_4, y_5
Then $G\left(K(f)\Big/_K\right) \cong \mathbf{S}_5$, and we have known that $\mathbf{S}_5$ is solvable.
Hence $K(f)$ is not a radical extension of K. □