EXAMPLE IN THE CLASS

Example: $p \equiv 3(mod\ 4)$, show that $x^2 \equiv -1(mod\ p)$ has no solution.
proof: If $a^2 \equiv -1(mod\ p)$ then $a^4 = 1$ in $\mathbb{Z}_p$
Hence $|a| = 4 \mid |U_p| = p - 1$, constradication.

Example: $p \equiv 3(mod\ 4)$ is a prime. Show that $p \neq a^2 + b^2$ for $a, b \in \mathbb{Z}$.
proof: If $p = a^2 + b^2$ for some $a, b \in \mathbb{Z}$. Then $a^2 + b^2 \equiv 0(mod\ p)$
Note $a \neq 0(mod\ p)$.
Hence $1 + (b/a)^2 \equiv 0(mod\ p)$.ie. $b/a$is a solution of $x^2 - 1(mod\ p)$, constradiction.

Example: For $a, b, c, d \in \mathbb{Z}$, show that $(a^2 + b^2)(c^2 + d^2) = e^2 + f^2$ for some $e, f \in \mathbb{Z}$
proof:Choose $e + fi = \overline{(a + bi)}(c + di)$
Then $e^2 + f^2 = (e + fi)\overline{(e + fi)} = (a + bi)(c + di)\overline{(a + bi)(c + di)} = (a^2 + b^2)(c^2 + d^2)$