

進階代數(下) 第一次作業

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1. (邱鈺傑) If G is a finite group of even order, then G contains an element $a \neq e$ such that $a^2 = e$.
2. (蕭雯華) If p is a prime, then the nonzero elements of \mathbb{Z}_p form a group of order $p - 1$ under multiplication.

以下是建模問題

3. (王信元) A machine accepts eight-letter words, and prints an eight-letter word consisting of the first five letters of the first word followed by the last three letters of the second word. Show that the set of eight-letter words with this composition is a semigroup. Is this a monoid.
4. (陳巧玲) A machine accepts eight-letter words, and prints an eight-letter word consisting of the last four letters of the first word followed by the first four letters of the second word. Is the set of eight-letter words with this composition a semigroup?
5. Let $P_3 = \{1-2-3\}$ be a path. Color each vertex in P_3 by black(off) or white(on) to have 8 configurations. By a *move* associated with a vertex $i \in P_3$ we mean a function that sends each configuration to another configuration by changing the colors of the neighbors of i . By a *plus move* associated with a vertex $i \in P_3$ we mean we mean a function that sends each configuration to another configuration by changing colors of the neighbors of i and also changing the color of i . A *lit-only move* (resp. *lit-only plus move*) associated with an element $i \in P_3$ is similar to a move (resp. plus move) associated with i except that it does nothing if the configuration has black color in i . By a *dual lit-only move* associated with a vertex $i \in P_3$ we mean a function on the set of configurations that changes no colors of any vertices except i and only in the case that i has odd number of white neighbors to change the color of i .
 - (a) (林詒琪) Show that the three moves in P_3 generates a group \mathbf{M} . What is the order of \mathbf{M} .
 - (b) (葉彬) Show that the three plus moves in P_3 generates a group \mathbf{M}^+ . What is the order of \mathbf{M} .
 - (c) (張英峰) Interpret the lit-only moves properly such that the three lit-only moves in P_3 generate a group \mathbf{L} .
 - (d) (林育生) Show that \mathbf{L} is not abelian.
 - (e) (黃彥璋) Show that the lit-only plus moves in P_3 generate a semigroup \mathbf{L}^+ , but not a group.
 - (f) (林志嘉) Interpret the dual lit-only moves properly such that the three dual lit-only moves in P_3 generate a group \mathbf{L}^* .
 - (g) (陳建文) Show that \mathbf{L}^* is not abelian.