

進階代數(下) 第十次作業

上課老師: 翁志文

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- Let n be a positive integer and let $U_n := \{m \mid 0 \leq m \leq n-1, (m, n) = 1\}$.
 - (賴德展) How many elements in U_{100} ?
 - (劉俞欣) Show that U_n is a group under multiplication modulo n .
 - (周彥伶) Let p be a prime. Show that $a^p \equiv a \pmod{p}$.
 - (洪湧昇) Let $q < p$ be primes and $q \mid p-1$. Suppose that you have learned the group U_p to be cyclic. Show that there exists an integer s such that $s \not\equiv 1 \pmod{p}$ and $s^q \equiv 1 \pmod{p}$.
- Let $q < p$ be two primes and $q \mid p-1$. Let s be an integer satisfying 1(d).
 - (林志峰) Show that the map $\alpha : \mathbb{Z}_p \rightarrow \mathbb{Z}_p$ given by $\alpha(i) = si$ is an additive group automorphism.
 - (黃正一) Show that the map $\theta : \mathbb{Z}_q \rightarrow \text{Aut } \mathbb{Z}_p$ given by $\theta(i) = \alpha^i$ is a homomorphism.
 - (邱鈺傑) Let $\mathbb{Z}_p \rtimes_{\theta} \mathbb{Z}_q$ be the semidirect product of \mathbb{Z}_p and \mathbb{Z}_q as defined in Homework 7. Show that $|(1, 0)| = p$, $|(0, 1)| = q$ and $(0, 1)(1, 0) = (s, 0)(0, 1)$. $\mathbb{Z}_p \rtimes_{\theta} \mathbb{Z}_q$ is called the *metacyclic group*.
- (蕭雯華) (Normalizer grows in p -group) Let $|G| = p^n$ where p is a prime, $H < G$ and $H \neq G$. Show that $H \neq N_G(H)$.
- (陳巧玲) Suppose $|G| = p^n$ and $\langle e \rangle \neq H \triangleleft G$. Show $H \cap Z(G) \neq \langle e \rangle$.
- (林詒琪) Let $|G| = p^n$. Show that for each $0 \leq k \leq n$, G has a normal subgroup of order p^k .
- (葉彬) Let H be a normal subgroup of order p^k of a finite group G . Show that H is contained in every Sylow p -subgroup of G .