進階代數(下) 第十一次作業

上課老師: 翁志文

2009 年五月四日

1. (林育生) If $H \in \mathrm{Syl}_p(G)$ and $H \triangleleft G$, then $\mathrm{Syl}_p(G) = \{H\}$.
2. (黃彥璋) If $|G| = p^n q$, with $p > q$ primes, then G contains a unique normal Sylow subgroup of index q.
3. (林志嘉) Every group of order 12, 28, 56, and 200 must contain a normal Sylow subgroup, and hence is not simple.
4. (陳建文) How many elements of order 7 are there in a simple group of order 168?
5. (a) (羅健峰) Every group G of order p^n (p a prime) has nontrivial center.
   (b) (黃思綸) Every group G of order p^2 (p a prime) is abelian.
6. (a) (陳泓勳) The transpositions $(1,2)$, $(2,3)$, $\ldots$, $(n, n+1)$ generate the symmetric group $\mathbf{S}_{n+1}$.
   (b) (何昕暘) Construct an epimorphism from the Coxeter group W associated with P_n to the group $\mathbf{L}$ generated by lit-only moves in P_n as described in homework 1.
   (c) (賴德展) Let S_i denote the lit-only move associated with $i \in P_n$, interpreted as an $n \times n$ matrix over $\mathbb{Z}_2$. Let $e_1 = (1, 0, 0, \ldots, 0)^t$. Set $S = \{e_1, S_1 e_1, S_2 S_1 e_1, \ldots, S_n S_{n-1} \cdots S_1 e_1\}$. Let $\mathbf{L} =< S_1, S_2, \ldots, S_n >$ denote the subgroup generated by S_i for $1 \le i \le n$. Find an action of $\mathbf{L}$ on S.
   (d) (劉侖欣) Find a homomorphism of $\mathbf{L}$ into $\mathbf{S}_{n+1}$ corresponding to the above action.
   (e) (周彥伶) *Determine the Coxeter group associated with P_3.