Homework 2.

1.

(a) $a = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $a^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $a^3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\Rightarrow$ a is order of 4.

$b = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$, $b^2 = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$

$b^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ b is order of 3.

(b) $ab = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $(ab)^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ ab is order of 0.

2. $G = \bigcup_{a \in G}$

case 1.

$| < a > | < \infty$ for any $a \in G \Rightarrow$ G is finite.

case 2.

$| < a > | = \infty$ for some $a \in G$

$\Rightarrow | < a > | \cong (Z, +)$ (contract)

$\Rightarrow$ G is finite.

3.$(H \supseteqq K)$

$\forall k \in K$, $\exists h \in H$, $n, n' \in N$

s.t $hn = kn'$ since $HN = KN$

$\Rightarrow kh' = nn'^{-1} \in N$

Since $H < K$, $kh^{-1} \in K$,

thus $kh^{-1} \in K \cap N = H \cap N \subseteq H$.

So $k \in H \Rightarrow K \subseteq H$ and $H < K$, then $H = K$

4. If $N < G$, $[G : N] = 2$

Claim: $N \lhd G$

$\forall a \in G$, $a \notin G$

$\Rightarrow a \in aN$, $a \in Na$(left coset and right coset)

$\Rightarrow aN = Na \Rightarrow N \lhd G$