# 進階代數(下) 第三次作業

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1. (蕭雯華) Show that a group of order $pq$ has at most one subgroup of order $p$, where $p > q$ are primes.

2. (陳巧玲) Let $G$ be the group of all nonzero complex numbers under multiplication and let $N$ be the set of complex numbers of absolute value 1. Show that $G/N$ is isomorphic to the group of all positive real numbers under multiplication.

3. (林詒琪) Let $G$ be the group of real numbers under addition and let $N$ be the subgroup of $G$ consisting of all the integers. Prove that $G/N$ is isomorphic to the group of all complex numbers of absolute value 1 under multiplication.

4. (葉彬) Prove that every finite group having more than two elements has a nontrivial automorphism.

5. (林育生) Prove that any element $\sigma \in S_n$ which commutes with $(1, 2, \ldots, r)$ is of the form $\sigma = (1, 2, \ldots, r)^i \tau$ for some $\tau \in S_n$ with $\tau(i) = i$ for all $1 \leq i \leq r$.

6. Two elements $a, b \in G$ are *conjugate* if there exists $c \in G$ such that $a = c^{-1}bc$. The conjugate is an equivalent relation on $G$ and hence $G$ is partitioned into *conjugate classes*.
   (a) (黃彥璋) Find the number of conjugates of $(1, 2)(3, 4)$ in $S_n$, $n \geq 4$.
   (b) (林志嘉) Find the form of all elements commuting with $(1, 2)(3, 4)$ in $S_n$, $n \geq 4$.
   (c) (陳建文) Determine number the conjugate classes of $S_6$.

(建模問題)

7. Let the column vector $u = (u_1, u_2, u_3)^t$ represent a coloring configuration of the path $P_3 = \{1 - 2 - 3\}$ described in Homework 1, where $u_i \in \mathbb{Z}_2$; $u_i = 0$ iff the vertex $i$ is colored in black (off).
   (a) (羅健鋒) Interpret each lit-only move associated with a vertex $i$ "faithfully" to a $3 \times 3$ matrix $S_i$ with entries over $\mathbb{Z}_2$ such that $S_i$ sends a configuration $u$ to $S_i u$.
   (b) (陳泓勳) Interpret each dual lit-only move associated with a vertex $i$ "faithfully" to a $3 \times 3$ matrix $S_i^*$ with entries over $\mathbb{Z}_2$ such that $S_i^*$ sends a configuration $u$ to $S_i^* u$. What is the relation between $S_i$ and $S_i^*$.
   (c) (何欣暘) Interpret each dual lit-only plus move associated with a vertex $i$ "faithfully" to a $3 \times 3$ matrix $M_i$ with entries over $\mathbb{Z}_2$ such that $M_i$ sends a configuration $u$ to $M_i u$.
   (d) (賴德展) *Let $\mathbf{L} =< S_1, S_2, S_3 >$ . Show that the center $Z(\mathbf{L})$ of $\mathbf{L}$ is trivial. (Hint. Compute $S_i S$ and $S S_i$ if there exists $S \in Z(\mathbf{L})$ with $S_{ij} = 1$.)