

進階代數(下) 第四次作業

上課老師: 翁志文

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1. (黃思綸) Let $H, N \triangleleft G$, $HN = G$ and $H \cap N = \{e\}$. Show that G is isomorphic to $H \times N$.
2. (劉侖欣) Let $G_1, G_2, G_3 \triangleleft G$ such that $G_1G_2G_3 = G$ and $G_i \cap G_jG_k = \{e\}$ for any set $\{i, j, k\} = \{1, 2, 3\}$. Show that G is isomorphic to $G_1 \times G_2 \times G_3$.
3. (周彥伶) Show that S_3 is not the direct product of any family of its proper subgroups.
4. (洪湧昇) Show that \mathbb{Z}_{p^n} is not the direct sum of any family of its proper subgroups, where p is a prime.
5. (林志峰) Show that \mathbb{Z} is not the direct sum of any family of its proper subgroups.
6. (黃正一) Is it possible for a cyclic group to be a direct product of its two proper subgroups.
7. (邱鈺傑) If G is a group and $N \triangleleft G$, show that if $a \in G$ has finite order $|a|$, then Na in G/N has finite order m , where m divides $|a|$.
8. Let G be a finite group, α an automorphism of G , and set

$$I = \{g \in G \mid \alpha(g) = g^{-1}\}.$$

- (a) (蕭雯華) Suppose $|I| > \frac{3}{4}|G|$. Show that G is abelian. (Hint. $I \cap h^{-1}I \subseteq C_G(h)$ for $h \in I$.)
- (b) (陳巧玲) Suppose $|I| = \frac{3}{4}|G|$. Show that G has an abelian subgroup of index 2. (Hint. Consider $C_G(h)$ for $h \in I - Z(G)$.)