

進階代數(下) 第五次作業

上課老師: 翁志文

2009 年三月十九日

1. (林詒琪) Let H be a proper subgroup of a finite group G . Show that $G \neq \bigcup_{g \in G} gHg^{-1}$.
(Hint. Consider the normalizer of H)
2. Let X be a set of symbols, $F(X)$ be the free group generated, $Y \subseteq F(X)$, and N be the smallest normal subgroup containing Y .
 - (a) (葉彬) Show that $N = \langle \{g^{-1}yg \mid g \in F(X), y \in Y\} \rangle$.
 - (b) (林育生) *For $X = \{x, y\}$ and $Y = \{y\}$, show that $F(X)/N$ is isomorphic to $F(\{x\})$.
3. Let $P_n = \{1 - 2 - \cdots - n\}$ be a path. The *Coxeter group* W associated with P_n is a group with n generators s_1, s_2, \dots, s_n subject only to the following product rules:

$$(s_i s_j)^{m(s_i, s_j)} = e,$$

where

$$m(s_i, s_j) = \begin{cases} 1, & i = j; \\ 2, & i, j \text{ are not adjacent}; \\ 3, & i, j \text{ are adjacent}. \end{cases}$$

- (a) (黃彥璋) Construct an epimorphism from the Coxeter group W associated with P_3 to the group \mathbf{L} generated by lit-only moves in P_3 as described in homework 1.
- (b) (林志嘉) Determine the Coxeter group associated with P_1 .
- (c) (陳建文) Determine the Coxeter group associated with P_2 .
- (d) (羅健峰) Determine the group \mathbf{L} generated by lit-only moves in P_2 .