進階代數(下) 第六次作業

上課老師: 翁志文

2009 年三月二十六日

1. (黃思綸) Show that no group is the union of its two proper subgroups.
2. (陳泓勳) Find an abelian group G and a subgroup H of G such that $G \neq H \times K$ for all subgroups K of G.
3. Let p be prime and let $\mathbb{Q}/\mathbb{Z}$ be the additive quotient group of $\mathbb{Q}$ by $\mathbb{Z}$. Set
$$\mathbb{Z}(p^\infty) = \{\overline{a/p^i} \in \mathbb{Q}/\mathbb{Z} \mid a \in \mathbb{Z},\ i \in \mathbb{N} \cup \{0\}\}.$$
   (a) ((何欣暘)) Show that $\mathbb{Z}(p^\infty)$ is a subgroup of $\mathbb{Q}/\mathbb{Z}$.
   (b) (賴德展) Show that every element of $\mathbb{Z}(p^\infty)$ has finite order p^n for some $n \geq 0$.
   (c) (劉倫欣) Determine all the subgroups of $\mathbb{Z}(p^\infty)$.
   (d) (周彥伶) Show that $\mathbb{Z}(p^\infty)$ satisfies descending chain condition, but does not satisfy ascending chain condition on normal subgroups.
   (e) (洪湧昇) Write $\mathbb{Z}(p^\infty)$ as a finite direct sum of indecomposable subgroups.
4. (林志峰) Show that the additive group $\mathbb{Q}$ is indecomposable.
5. (黃正一) Write the additive group $\mathbb{Z}_{150}$ as a finite direct sum of indecomposable subgroups.