進階代數(下) 第七次作業

上課老師: 翁志文

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1. Suppose $G = H \times K$.
   (a) (邱鈺傑) Let N be a normal subgroup of H. Show that N is normal in G.
   (b) (蕭雯華) Suppose that G satisfies the ACCN. Show that H satisfies ACCN.
   (c) (陳巧玲) Suppose that G satisfies the DCCN. Show that H satisfies DCCN.

2. Let Q_8 be the multiplication group generated by the complex matrices

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

   Q_8 is called the *quaternion group.*
   (a) (林詒琪) Show $|Q_8| = 8$.
   (b) (葉彬) *Consider the set $G = \{\pm 1, \pm i, \pm j, \pm k\}$ with multiplication given by $i^2 = j^2 = k^2 = -1$; $ij = k = -ji$; $jk = i = -kj$, $ki = j = -ik$, and the usual rules for multiplying by ± 1. Show that G is a group isomorphic to the quaternion group Q_8.
   (c) (林育生) What is the center $Z(Q_8)$ of the quaternion group Q_8?
   (d) (黃彥璋) Show that $Q_8/Z(Q_8)$ is abelian.
   (e) (林志嘉) Is Q_8 isomorphic to $(Z(Q_8) \times Q_8/Z(Q_8))$.

3. Let $N \lhd G$, $H < G$, $N \cap H = \{e\}$ and $G = HN$. In this case G is called the *inner semidirect product* of N and H. For each $h \in H$, define a map $\phi_h : N \to N$ by $\phi_h(n) = hnh^{-1}$ for $n \in N$.
   (a) (陳建文) Is it always true $G = N \times H$.
   (b) (羅建峰) Show that ϕ_h is an automorphism on N.
   (c) (黃思綸) Show that the map $\phi : H \to \mathrm{Aut}(N)$, defined by $\phi(h) = \phi_h$, is a homomorphism from H into $\mathrm{Aut}(N)$.
   (d) (陳泓勳) Show that $nhn'h' = n\phi_h(n')hh'$ for $n, n' \in N$ and $h, h' \in H$.
   (e) (何欣暘) Write D_8 as an inner semidirect product of its two nontrivial subgroups.

4. Let N, H be groups and $\phi : H \to \mathrm{Aut}(N)$ a homomorphism. Let $N \rtimes_\phi H$ be the set $N \times H$ with the following binary operation:

$$(n, h)(n', h') = (n\phi(h)(n'), hh')$$

   for $n, n' \in N$ and $h, h' \in H$.
   (a) (賴德展) Is there an identity for the above operation?
   (b) (劉侖欣) Does there exist $(n, h)^{-1}$ for each $(n, h) \in N \rtimes_\phi H$?
   (c) (周彥伶) Show that $N \rtimes_\phi H$ is a group with the above operation. $N \rtimes_\phi H$ is called the *outer semidirect product* of N and H.