進階代數(下) 第八次作業

上課老師: 翁志文

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1. Let G be a finite abelian group.
   (a) (洪湧昇) Suppose $G = \mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_9$. Find subgroups $H, K < G$ of orders 36, 2 respectively such that $G = H \times K$.
   (b) (林志峰) Suppose $G = \mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_9$. Find subgroups $H, K < G$ of orders 36, 2 respectively such that $G \neq H \times K$.
   (c) (黃正一) Suppose $G = \mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_9$. Find a subgroup $H < G$ of order 36 such that $G \neq H \times K$ for any subgroup K of G.
   (d) (邱鈺傑) *Show that if G is not cyclic, then G is decomposable. (Hint. $G =< a > \times K$ for some $a \in G$.)
   (e) (蕭雯華) Show that G is an inner direct product of a finite number of indecomposable subgroups. What groups are these subgroups isomorphic to?
   (f) (陳巧玲) Is it possible that $G = G_1 \times G_2 = H_1 \times H_2$, where $G_1 \cong G_2 \cong Z_4$, $H_1 \cong Z_2$ and $H_2 \cong Z_8$?
   (g) (林詒琪) Use Krull-Schmidt Theorem to determine how many non-isomorphic abelian groups of order 1400.