進階代數(下) 第九次作業

上課老師: 翁志文

2009 年四月十六日

1. (葉彬) Find an indecomposable group G, a decomposable group H and an epimorphism $\phi : G \to H$.

2. The following fact might be used in doing this set of problems: The alternating group A_n is simple if and only if $n \neq 4$.

   (a) (林育生) Show that A_4 is indecomposable.

   (b) (黃彥璋) Find groups N, H, K such that $N \subseteq H \times K$, $N \cap (H \times \{e'\}) = \{(e, e')\}$ and $N \cap (\{e\} \times K) = \{(e, e')\}$.

   (c) (林志嘉) *Suppose $G = H \times K$ and $N \lhd G$. Show that $N < Z(G)$, $N \cap H \neq \{e\}$ or $N \cap K \neq \{e\}$, where $Z(G)$ is the center of G. (Hint. 不要想, 純邏輯推演)

   (d) (陳建文) *Show that S_n is indecomposable for $n \geq 2$.

3. (羅健峰) *Show that any finite group is isomorphic to a subgroup of A_n for some n. (Hint. Let G act on two copies of G by left translations)

4. (黃思綸) *Let H be a proper subgroup of G and the index of H in G finite. Show that G contains a proper normal subgroup of finite index. (Hint. Let G act on left cosets of H.)

5. (陳泓勳) *Suppose $|G| = pn$, with $p > n$, p prime, and $|H|$ is a subgroup of order p. Show $H \lhd G$. (Hint. Let G act on left cosets of H.)

6. (何昕暘) *Suppose $|G| = p^n$, and $N \lhd G$ with $|N| = p$ a prime. Show that $N \subseteq Z(G)$. (Hint. Let G act on N by conjugation)