

# 微積分B(二) 第一次作業

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請同學以活頁紙整齊且詳細作答後用訂書機訂起來,  
於三月二十日課堂上交給我

1. Find the sum of the series

(a)  $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+3)}$ ;

(b)  $\sum_{k=0}^{\infty} \frac{(-5)^k}{4^{k+1}}$ .

2. Show that

$$\sum_{k=1}^{\infty} kx^{k-1} = \frac{1}{(1-x)^2}$$

for  $|x| < 1$ . (Hint: 高中數學)

3. Determine whether the series converges or diverges.

(a)  $\sum \frac{\ln k}{k^2 + 1}$ ;

(b)  $\sum \frac{7}{k(\ln k)^2}$ ;

(c)  $\sum \frac{k}{1 + 2^2 + 3^2 + \dots + k^2}$ ;

(d)  $\sum \frac{1}{(\ln k)^{10}}$ ;

(e)  $\sum \frac{k!}{k^k}$ ;

(f)  $\sum (\sqrt{k} - \sqrt{k-1})^k$ ;

(g)  $\frac{2}{3} + \frac{2 \cdot 4}{3 \cdot 7} + \frac{2 \cdot 4 \cdot 6}{3 \cdot 7 \cdot 11} + \dots$ ;

(h)  $\frac{1}{2 \ln 2} - \frac{1}{3 \ln 3} + \frac{1}{4 \ln 4} - \frac{1}{5 \ln 5} + \dots$ ;

(i)  $\frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} - \frac{1}{7} + \frac{1}{8} - \frac{1}{9} - \frac{1}{10} + \dots$ ;

(Hint: 用定義或其它我不知道的巧妙方法)

(j)  $\sum \left(1 - \frac{1}{k}\right)^k$ ;

4. Determine whether the series converges or diverges.

(a)  $\sum a_n$ , where

$$a_n = \begin{cases} \frac{1}{k}, & \text{if } n = 10^k; \\ 0, & \text{else.} \end{cases}$$

(b)  $\sum (-1)^n b_n$ , where

$$b_n = \begin{cases} \frac{1}{n} + \frac{1}{k}, & \text{if } n = 10^k; \\ \frac{1}{n}, & \text{else.} \end{cases}$$

5. (挑戰題考古題) Let  $\sum_{n=1}^{\infty} a_n$  be a convergent sequence, where  $a_n > 0$  for all  $n = 1, 2, \dots$ .

(a) Is  $\sum_{n=1}^{\infty} (e^{a_n} - 1)$  convergent? Show your work;

(b) Is  $\sum_{n=1}^{\infty} (e^{a_n} - 1) \ln(1 + \frac{1}{n^2})$  convergent? Show your work.