微積分(二) 第三次作業

上課老師: 翁志文

2009 年三月三十一日

請同學以活頁紙整齊且詳細作答後用訂書機訂起來,
於四月十日課堂上交給我. 四月二十一日期中考.

1. Explain that for any $a \in \mathbb{R}$ and any sequence $\{c_n\}$, $[5, \infty)$ is not the interval of convergence of the power series $\sum\limits_{n=0}^{\infty} c_n(x-a)^n$.

2. Suppose $\sum\limits_{n=0}^{\infty} c_n(-3)^n$ converges. Show that $\sum\limits_{n=0}^{\infty} c_n 2^n$ converges.

3. Find the radius of convergence and interval of convergence of the series.

   (a) $\sum\limits_{n=0}^{\infty} \dfrac{n^2 x^n}{10^n}$;

   (b) $\sum\limits_{n=1}^{\infty} \dfrac{n x^n}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$;

   (c) $\sum\limits_{n=2}^{\infty} (-1)^n \dfrac{(2x+3)^n}{n \ln n}$.

4. A function f is defined by
   $$f(x) = 1 + 2x + x^2 + 2x^3 + x^4 + \cdots$$

Find the interval of convergence the series and find an explicit formula for $f(x)$.

5. Find a power series representation for the function and determine the radius of convergence.

   (a) $f(x) = \ln(5 - x)$;

   (b) $f(x) = x \ln(1 + x)$;

6. Evaluate the indefinite integral as a power series.

   (a) $\int \dfrac{1}{1 + x^4}$;

   (b) $\int \tan^{-1}(x^2) dx$.

7. Evaluate $\int \dfrac{\sin x}{x} \, dx$ as an infinite series.

8. Find the sum of the series
   $$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots$$