

微積分(二) 第四次作業

上課老師: 翁志文

2009 年四月二十八日

請同學以活頁紙整齊且詳細作答後用訂書機訂起來,
於五月八日課堂上交給我

1. Find the limit, if it exist, or show that the limit does not exist.
 - (a) $\lim_{(x,y) \rightarrow (5,-2)} (x^5 + 4x^3y - 5xy^2)$;
 - (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2+y^2}}$; (Hint: 用極座標解釋)
 - (c) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$;
2. Find all the second partial derivatives of $f(x, y) = x^4 - 3x^2y^3$.
3. Determine whether each of the following functions is a solution of *Laplace's equation* $u_{xx} + u_{yy} = 0$.
 - (a) $u = x^2 + y^2$;
 - (b) $u = x^2 - y^2$.
4. Find the gradient vector of $f(x, y, z) = x - \sqrt{y^2 + z^2}$ at $(2, -3, 4)$.
5. Find a function f whose gradient is $\mathbf{F}(x, y) = (x + \sin y)\mathbf{i} + (x \cos y - 2y)\mathbf{j}$.
6. Find an equation of the tangent plane to the surface $z = y \ln x$ at the point $(1, 4, 0)$, and use it to approximate the value $4.01 \ln 0.99$.
7. Find the differential dz of $z = x^3 \ln(y^2)$.
8. $\ln(x + yz) = 1 + xy^2 + z^3$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
9. Find $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$, where $u = x^2 - xy + z^2$, $x = s \cos t$, $y = \sin(t - s)$, $z = s^2 + t^2$.