微積分(二) 第五次作業

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五月二十二日課堂上交.

1. Find the directional derivative of the function $f(x, y, z) = \dfrac{x}{y+z}$, at $(4, 1, 1)$ in the direction $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

2. Find the maximum rate change of $f(x, y, z) = x + \dfrac{y}{z}$ at $(4, 3, -1)$.

3. Find the tangent plane of $x = y^2 + z^2 - 2$, at the point $(-1, 1, 0)$.

4. Find the local maximum and minimum and saddle points of the function $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$.

5. Find the absolute maximum and minimum values of $f(x, y) = x^2 + 2xy + 3y^2$, on the closed triangular region with vertices $(-1, 1)$, $(2, 1)$, $(-1, -2)$.

6. Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid $9x^2 + 36y^2 + 4z^2 = 36$.

7. Find the maximum and minimum values of the function $f(x, y, z) = x^2y^2z^2$ subject to the constraint $x^2 + y^2 + z^2 = 1$.

8. Find the maximum and minimum values of $f(x, y, z) = 3x - y - 3z$, subject to the two constraints $x + y - z = 0$ and $x^2 + 2z^2 = 1$.

9. Evaluate the following double integral:

   (a) $\displaystyle\iint_{\Omega} x^2\, dxdy, \quad \Omega = \{(x, y) \mid 0 \le x \le 1,\ 0 \le y \le 3\};$

   (b) $\displaystyle\iint_{\Omega} \sin(x + y)\ \ dxdy, \quad \Omega = \{(x, y) \mid 0 \le x \le \tfrac{\pi}{2},\ 0 \le y \le \tfrac{\pi}{2}\}.$