

微積分(二) 第六次作業

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六月五日課堂上交.

1. Evaluate $\iint_{\Omega} (x + 3y^3) dx dy$, $\Omega: 0 \leq x^2 + y^2 \leq 1$.
2. Evaluate $\int_0^1 \int_y^1 e^{y/x} dx dy$.
3. Find the volume of the solid in the first octant bounded above by $z = x^2 + 3y^2$, below by the xy -plane, and on the sides by the cylinder $y = x^2$ and the plane $y = x$.
4. *Let Ω be the triangle with vertices $(0, 0)$, $(0, 1)$, $(1, 1)$. Show that

$$\text{if } \int_0^1 f(x) dx = 0 \quad \text{then} \quad \iint_{\Omega} f(x)f(y)dA = 0. \quad (\text{Hint: using symmetry})$$

5. Find the area of the region enclosed by the lemniscate $r^2 = 4 \cos 2\theta$.
6. A cylindrical hole of radius 3 is drilled through the center of a sphere of radius 5. Determine the volume of the material that has been removed from the sphere.
7. Find the area of the part of the surface $z = x + y^2$ that lies above the triangle with vertices $(0, 0)$, $(1, 1)$, $(0, 1)$.
8. Find the area of the part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.
9. Use triple integration to find the volume of the solid E bounded above by the parabolic cylinder $z = 4 - y^2$ and bounded below by the elliptic paraboloid $z = x^2 + 3y^2$.
10. Find the volume of the solid E bounded above by the plane $z = y$ and below by the paraboloid $z = x^2 + y^2$.

11. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^2 \sin(x^2 + y^2) dz dy dx$.

12. Find the Jacobian of the transformation $x = (1 + w \cos v) \cos u$, $y = (1 + w \cos v) \sin u$, $z = w \sin v$.

13. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{x^2 + y^2 + z^2} dz dy dx$.

14. Evaluate $\iint_{\Omega} (x^2 - y^2) dx dy$, where Ω is the parallelogram bounded by the four lines:
 $x + y = 0$, $x + y = 1$, $x - y = 0$, $x - y = 2$.