

# Partially Distance-regular Graphs and Partially Walk-regular Graphs\*

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## Abstract

We study partially distance-regular graphs and partially walk-regular graphs as generalizations of distance-regular graphs and walk-regular graphs respectively. We conclude that the partially distance-regular graphs can be viewed as some extremal graphs of partially walk-regular graphs. In the special case that the graph is assumed to be regular with four distinct eigenvalues, a well known class of walk-regular graphs, we show that there exists a rational function  $f$  in the expression of the order and the four eigenvalues of the graph such that  $k_2(x)$ , the number of vertices with distance 2 to a vertex  $x$ , satisfies  $k_2(x) \geq f$ ; furthermore we show the equality holds for each vertex  $x$  if and only if the graph is distance-regular with diameter 3.

Keywords: Partially distance-regular graphs; partially walk-regular graphs, eigenvalues.

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