AUTOMORPHISMS OF CERTAIN DESIGN GROUPS

ABSTRACT. Let N be an additive group and $\alpha \in \operatorname{Aut} N$. We say that α is *fixed point free* if $\alpha = \mathrm{id}$ or $\alpha(x) = x$ implies that $x = 0$. A group of automorphisms of N, $\Phi \leq \operatorname{Aut} N$ say, is called *regular* if every element of Φ is fixed point free. In this case, (N, Φ) is referred to as a *Ferrero pair*.

Let (N, Φ) and (N', Φ') be Ferrero pairs. An isomorphism $f : N \to N'$ is called an *equivalence* or an *isomorphism* between these Ferrero pairs if $\Phi' = f \Phi f^{-1}$. In case $N = N'$, this means of course that Φ and Φ' are conjugate in $\mathrm{Aut} N$.

Take a Ferrero pair (N, Φ) with finite N and nontrivial Φ. Customary, we set $v = |N|$ and $k = |\Phi|$, and denote $N^* = N \setminus \{0\}$. For $a \in N$, we write $\Phi\, a$ for the orbit of Φ in N, namely, $\Phi\, a = \{\varphi(a) \mid \varphi \in \Phi\}$. Then it holds that

(1) $k \,\big|\, (v - 1)$;
(2) $\{\Phi\, a \mid a \in N^*\}$ is a partition of N^* with $|\Phi\, a| = |\Phi| = k$ for all $a \in N^*$;
(3) for $a \in N^*$ and $b \in N$, $\Phi\, a + b = \Phi\, a$ implies that $b = 0$; and finally,
(4) if $\mathbf{S} = \{\Phi\, a_i \mid i = 1, \ldots, (v-1)/k\}$, where $\{a_i \mid i = 1, \ldots, (v-1)/k\}$ is a complete set of orbit representatives of Φ, then $\mathbf{S}$ is a $S_{k-1}(2, k; v)$ difference family. Thus, if $\mathbf{B}_\Phi = \{\Phi_{a_i} + b \mid 1 \leq i \leq (v-1)/k,\, b \in N\}$, then $(N, \mathbf{B}_\Phi)$ is a 2-design.

Given a Ferrero pair (N, Φ) with finite N, $(N, \mathbf{B}_\Phi, +)$ is a *design group*. Here, a structure $(N, \mathbf{B}, +)$ is called a design group if $(N, \mathbf{B})$ is a design and $(N, +)$ is a group such that the mapping $Q : N \to N$; $x \mapsto x + a$ is an automorphism of the design for each $a \in N$. Let $(N, \mathbf{B}, +)$ and $(N', \mathbf{B}', +)$ be two design groups. A mapping $N \to N'$ is called an isomorphism of the design groups, if it is at the same time an isomorphism of the groups and of the designs. If furthermore $(N, \mathbf{B}, +) = (N', \mathbf{B}', +)$, then it is called an automorphism.

Let (M, Ψ) and (N, Φ) be finite Ferrero pairs with M, N, Φ, and Ψ abelian, and let f be an isomorphism from the design group $(M, \mathbf{B}_\Psi, +)$ to the design group $(N, \mathbf{B}_\Phi, +)$. Put $k = |\Phi|$. If $|N| > k + 1$, then $f \Psi f^{-1} = \Phi$, i.e., f is an equivalence of the involved Ferrero pairs.

In this talk, we treat the case when Φ and Ψ are nonabelian under certain condition.

Theorem 1. *Let (M, Ψ) and (N, Φ) be finite Ferrero pairs and let f be an isomorphism from $(M, \mathbf{B}_\Psi, +)$ to $(N, \mathbf{B}_\Phi, +)$. Put $k = |\Phi|$. If $|N/[N, N]| > 2k^2 - 6k + 1$, then $f \Psi f^{-1} = \Phi$, i.e., f is an equivalence of the involved Ferrero pairs.*