On the Enumeration of Parking Functions by Leading Numbers

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Abstract

Let $\mathbf{x} = (x_1, \ldots, x_n)$ be a sequence of positive integers. An $\mathbf{x}$-parking function is a sequence $(a_1, \ldots, a_n)$ of positive integers whose non-decreasing rearrangement $b_1 \leq \cdots \leq b_n$ satisfies $b_i \leq x_1 + \cdots + x_i$. In this talk we give a combinatorial approach to the enumeration of $(a, b, \ldots, b)$-parking functions by their leading terms, which covers the special cases $\mathbf{x} = (1, \ldots, 1)$, $(a, 1, \ldots, 1)$, and $(b, \ldots, b)$. The approach relies on bijections between the $\mathbf{x}$-parking functions and labeled rooted forests. To serve this purpose, we present a simple method for establishing the required bijections. Some bijective results between certain sets of $\mathbf{x}$-parking functions of distinct leading numbers are also given. This talk is based on a joint work with Sen-Peng Eu and Chun-Ju Lai.