2020 FALL CALCULUS 0312: FIRST MIDTERM (NOVEMBER 20, 2020)

- Please answer the following questions in details, which means you need to state all theorems or results you used. The definitions of terminology were taught in the lectures, so you cannot ask instructor or TA about mathematical definitions while taking the midterm.
- Please mark your name, student ID, and question numbers clearly on your answer sheet.

The following theorem might be used to solve questions.

Theorem 0.1 (Mean Value Theorem)**.** *Let f be a continuous function on $[a,b]$, and f be differentiable on (a,b). Then there exists a number $c \in (a,b)$*

Theorem 0.2 (L'Hospital's rule)**.** *Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except for possibly at a). Suppose that*

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0,$$
$$\text{or that} \quad \lim_{x\to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x\to a} g(x) = \pm\infty.$$

Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right hand side exists (or is ∞ or $-\infty$).

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1. (10 points) With the same notations as in Theorem 0.1, let $f(x) = 2x^2 - 3x + 1$ and $[a,b] = [0,2]$. Verify that the function $f(x)$ satisfies the hypothesis of the Mean Value Theorem on the interval . Find the number c which satisfies the Mean Value Theorem, where c is the number given in Theorem 0.1.

2. (10 points) Prove the identity[1]
$$\arcsin \frac{x-1}{x+1} = 2\arctan\sqrt{x} - \frac{\pi}{2}.$$

3. (10 points) Find the local maximum and minimum values of $f(x) = \frac{x^2}{x-1}$ by using the second derivative test.

4. (10 points) For what values of c is the function
$$f(x) = cx + \frac{1}{x^2+3}$$
increasing on $\mathbb{R} = (-\infty, \infty)$?

[1] Hint: If f, g are two differentiable functions such that $f' = g'$, then $f = g + C$, for some constant C.

5. Find the following limits:
   (a) (10 points) $\lim_{x\to 0}(\csc x - \cot x)$.
   (b) (10 points) $\lim_{x\to 1}(2-x)^{\tan(\pi x/2)}$.

6. (10 points) If f' is continuous, $f(2) = 0$ and $f'(2) = 7$, evaluate

$$\lim_{x\to 0}\frac{f(2+3x)+f(2+5x)}{x}.$$

7. Let $f(x) = \frac{2x}{x^2+1}$ be a continuous function defined on $[1,3]$.
   (a) (10 points) Express $\int_1^3 f(x)\,dx$ by using the Riemann sum.
   (b) (10 points) Find the value in (a)[2].

8. (10 points) Evaluate the indefinite integral $\int \frac{\csc^2 x}{1+\cot x}\,dx$.

9. (10 points) Evaluate $\int_{-1}^{2}(x-2|x|)\,dx$.

10. (10 points) Evaluate $\lim_{x\to 3}\left(\frac{x}{x-3}\int_3^x \frac{\sin t}{t}\,dt.\right)$

Final Remark. The hints won't be appeared in the last midterm...

[2]Substitution law.