2020 FALL CALCULUS 0312: THIRD MIDTERM (DECEMBER 18, 2020)

- Please answer the following questions in details, which means you need to state all theorems or results you used.
- Please mark your name, student ID, and question numbers clearly on your answer sheet.

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1. (10 points) Find the volume of the solid obtained by rotating about the x-axis the region enclosed by the curves $y = \frac{9}{x^2+9}$, $y = 0$, $x = 0$, and $x = 3$.

2. (10 points) The rational number $\frac{22}{7}$ has been used as an approximation to the number π since the time of Archimedes. Show that
$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx = \frac{22}{7} - \pi.$$

3. (10 points) Evaluate the integral or show that it is divergent:
$$\int_0^1 \frac{\sqrt{\arctan x}}{1+x^2}\,dx.$$
(Hint: Try to use some substitution law with suitable trigonometric functions...)

4. (10 points) Find the values of c such that the area of the region bounded by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 576.

5. (a) (10 points) Show that
$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx.$$
(b) (5 points) Use (a) to evaluate $\int \cos^2 x\,dx$.
(c) (5 points) Use (a) and (b) to evaluate $\int \cos^4 x\,dx$.

6. (10 points) For $m, n \in \mathbb{N}$, show that $\int_{-\pi}^{\pi} \sin mx \cos nx\,dx = 0$.

7. (10 points) Find the surface area obtained by rotating the curve $y = x^3$, $0 \le x \le 2$ about the x-axis.

8. (20 points) Find the length of the curve
$$y = \int_1^x \sqrt{t^3 - 1}\,dt, \quad 1 \le x \le 4.$$

9. (20 points bonus) Show that
$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}.$$