

**2020 FALL REAL ANALYSIS (I) @ NCTU APPL. MATH.  
HOMEWORK 1**

- Please answer the following questions in details, which means you need to state all theorems and all reasons you have been using.
  - Please mark your name, student ID, and question numbers clearly on your answer sheet. The deadline to hand in the exercise is on September 24, 2020.
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- (1) Let  $R \subset \mathbb{R}^n$  be a *rectangle*, prove that  $m_*(R) = |R|$ .
- (2) Let  $E \subset \mathbb{R}$  be a set, the outer Jordan content is defined by

$$J_*(E) := \inf \left\{ \sum_{j=1}^J |Q_j| : E \subset \cup_{j=1}^J Q_j \right\}$$

where  $Q_j$ 's are intervals in  $\mathbb{R}$  for  $j = 1, 2, \dots, J$ .

- (a) Prove that  $J_*(E) = J_*(\overline{E})$ , where  $\overline{E}$  denotes the closure of  $E$ .
- (b) Construct an example such that  $J_*(E) = 1$  but  $m_*(E) = 0$ , where  $m_*(E)$  is the outer measure of  $E$ .
- (3) Let  $E_1, E_2 \subset \mathbb{R}$  be measurable sets. Show that  $E_1 \times E_2$  is a measurable set in  $\mathbb{R}^2$  and  $m(E_1 \times E_2) = m(E_1) \cdot m(E_2)$ <sup>1</sup>.

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<sup>1</sup>In this case we *define*  $0 \cdot \infty = 0$