2020 FALL REAL ANALYSIS (I) @ NCTU APPL. MATH.
HOMEWORK 5

- Please answer the following questions in details, which means you need to state all theorems and all reasons you have been using.
- Please mark your name, student ID, and question numbers clearly on your answer sheet. The deadline to hand in the exercise is on November 12, 2020.

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(1) Show that simple functions, step functions, and continuous functions with compact supports are dense in $L^1(\mathbb{R}^n)$, under the graph norm $\|\cdot\|_{L^1(\mathbb{R}^n)}$.

(2) Suppose that f is integrable, and define $f_h(x) = f(x - h)$, for $h \in \mathbb{R}^n$. Show that $\|f_h - f\|_{L^1(\mathbb{R}^n)} \to 0$ as $h \to 0$.

(3) For any $p > 0$, $\int_E |f - f_k|^p \to 0$ as $k \to \infty$, and $\int_E |f_k|^p \leq M$ for all $k \in \mathbb{N}$. Show that $\int_E |f|^p \leq M$.

(4) Let $y = Tx$ be a non-singular linear transformation of $\mathbb{R}^n$. Suppose that $\int_E f(y)\, dy$ exists. Show that[1]

$$\int_E f(y)\, dy = |\det T| \int_{T^{-1}E} f(Tx)\, dx.$$

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[1]This question is a makeup question of 3(b) in the previous exercise.