2020 FALL REAL ANALYSIS (I) @ NCTU APPL. MATH.
HOMEWORK 6

- Please answer the following questions in details, which means you need to state all theorems and all reasons you have been using.
- Please mark your name, student ID, and question numbers clearly on your answer sheet. The deadline to hand in the exercise is on November 19, 2020.

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(1) Let $A \subset \mathbb{R}^n$ be a Borel measurable set, and $\chi_A^* = \chi_{A^*}$ be the symmetric-decreasing rearrangement. Prove that χ_A^* is a lower semicontinuous function.

(2) Let $f : \mathbb{R}^n \to \mathbb{R}$ be a Borel function vanishing at infinity. The function $f^*(x) := \int_0^\infty \chi^*_{\{|f|>t\}}(x)\, dt$. Show that f^* is measurable and lower semicontinuous.

(3) Let f be nonnegative and measurable on E and $\omega(\alpha) := \omega_{f,E}(\alpha)$ be finite on $(0, \infty)$. Suppose that $\int_0^\infty \alpha^{p-1}\omega(\alpha)\, d\alpha$ is finite, show that $\lim\limits_{a\to 0^+} a^p\omega(a) = \lim\limits_{b\to\infty} b^p\omega(b) = 0$.