

**2020 FALL REAL ANALYSIS (I) @ NCTU APPL. MATH.  
HOMEWORK 7**

- Please answer the following questions in details, which means you need to state all theorems and all reasons you have been using.
  - Please mark your name, student ID, and question numbers clearly on your answer sheet. The deadline to hand in the exercise is on November 26, 2020.
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- (1) If  $f(x)$  is a measurable function on  $\mathbb{R}^n$ . Show that  $F(x, y) := f(x - y)$  is also measurable on  $\mathbb{R}^{2n}$ .
- (2) Let  $f$  be measurable and *periodic* with period 1:  $f(x+1) = f(x)$  for all  $x$ . Suppose that there exists a finite number  $c$  such that

$$\int_0^1 |f(a+x) - f(b+x)| dx \leq c$$

for any  $a$  and  $b$ . Show that  $f$  is integrable in  $[0, 1]$ .

- (3) Let  $f$  be integrable on  $(-\infty, \infty)$ , and let  $h > 0$  be fixed. Show that

$$\int_{-\infty}^{\infty} \left( \frac{1}{2h} \int_{x-h}^{x+h} f(y) dy \right) dx = \int_{-\infty}^{\infty} f(x) dx.$$