## 2020 SPRING CALCULUS 0412: THIRD MIDTERM (MAY 28, 2020)

- Please answer the following questions in details, which means you need to state all theorems and all reasons you used. If you only write the the answer of the question, you will not get grade. The definitions of terminology were taught in the lectures, so you cannot ask instructor or TA about mathematical definitions while taking the midterm.
- Please mark your name, student ID, and question numbers clearly on your answer sheet.

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1. (10 points) Let $f(x,y) = x^{2020} + 2020xy + y^{2020}$, and $(a,b)$ be the point where $f$ has a local minimum. Find the directional derivative of $f$ at $(a,b)$ in the direction $(1,1)$.
2. (10 points) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of $xy^2 + yz^2 + zx^2 = 2020$.
3. (10 points) Find the value of $\iint_R (x+\sqrt{3}y)^2\, dA$, where $R = \{(x,y) |\, x^2+xy+y^2 \le 1\}$.
4. Let $F(x,y,z) = (\cos y, z^2 - x\sin y, 2(y+1)z)$ be a vector-valued function defined in $\mathbb{R}^3$. Let $f = f(x,y,z)$ be a scalar function such that $\nabla f = F$.
   (a) (5 points) State the condition for the existence of $f$.
   (b) (10 points) Find the scalar function $f$.
5. (10 points) Let $f(x,y) = x^3 - 3x + y^4 - 2y^2$, find all critical points.
6. (10 points) State the Fubini's theorem.
7. Let $z = f(x,y)$ be a surface. Explain
   (a) (5 points) The level set of $z = f(x,y)$.
   (b) (5 points) The geometrical meaning of the gradient.
8. (10 points) Find the maximum value of $x^2 + 2y - z^2$ subject to the constraints $2x - y = 0$ and $y + z = 0$.
9. (15 points) Evaluate the surface integral $\iint_S (x^2+y^2)z\, d\sigma$, where $S$ is the part of the plane $z = 4 + x + y$ which lies inside the cylinder $x^2 + y^2 = 4$.
10. (20 points) Evaluate $\int_{-\infty}^{\infty} e^{-x^2}\, dx$ (you need to explain the detailed reasons in your answer).