# 2021 FALL MATHEMATICAL LOGIC AND REASONING 1372: HOMEWORK

- Please answer the following questions in details, which means you need to state all theorems or results you used.
- Please mark your name, student ID, and question numbers clearly on your answer sheet.
- This homework takes 20 percent of your final grade.
- There are **eight** questions of this homework.
- **The deadline of this homework is on December 30, 2021.**

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1. Let $\mathcal{P}$ be a partition of the nonempty set $A$. For $x, y \in A$, define $xQy$ if and only if there exists $C \in \mathcal{P}$ such that $x \in C$ and $y \in C$. Prove that
   (a) $Q$ is an equivalence relation on $A$.
   (b) $A/Q = \mathcal{P}$.
2. Let $m \in \mathbb{N}$, then show that
   (a) For integers $x, y$, $x = y (\mathrm{mod}\, m)$ if and only if the remainder when $x$ is divide by $m$ equals to remainder when $y$ is divided by $m$.
   (b) $\mathbb{Z}_m$ consists of $m$ distinct equivalence classes: $\mathbb{Z}_m = \left\{\overline{0}, \overline{1}, \overline{2}, \ldots, \overline{m-1}\right\}$.

- Read Hasse diagram by yourselves.

Remaining questions from the textbook: Section 3.2 **#19**; Section 3.3 **#14**; Section 3.4 **#9**; Section 3.5 **#14**
Extra questions (if I can teach Chapter 5): Section 5.1 **#21**; Section 5.3 **#14**.